BOUNDARY VALUE PROBLEMS ON LIPSCHITZ AND CHORD-ARC DOMAINS

Let $L = \text{div} A \nabla$ be a second order elliptic divergence form operator, that is, $A = (a_{i,j})$ is a matrix whose (real) coefficients have the property that there exists a $\lambda$ such that for all $\zeta_j$ with $\sum_j |\zeta_j|^2 = 1$,

$$\lambda < \sum_{i,j} a_{i,j} \zeta_i \zeta_j < (\lambda)^{-1}.$$

The Laplacian ($A$ is the identity) is the classical example. The Dirichlet problem is the problem of finding unique solutions to $Lu = 0$ in a given domain with prescribed boundary conditions. These problems have an interpretation even when the boundary of the domain is very irregular (Lipschitz, chord-arc) and when the boundary values $f$ belong to spaces of discontinuous functions ($L^p$), so that the sense in which solutions take on their boundary values is delicate.

In the 90’s, a rather complete theory of solvability of boundary value problems for operators which could be regarded as perturbations of the Laplacian was developed. A key tool is the equivalence between solvability of the Dirichlet problem with boundary data in $L^p$ and a Muckenhoupt condition on the measure induced by the elliptic operator. In joint work with M. Dindos and C. Kenig, we proved the endpoint (or $A^\infty$) version of this equivalence. And, recently, in joint work with A. Millakis and T. Toro, we extended the perturbation theory known for Lipschitz domains to chord-arc domains. Both of these investigations involve the function spaces $BMO$ and the associated notions of Carleson measures. The goal of this lecture is to explain the main ideas as well as the role of the geometry of the domains.