Math 5285 Honors abstract algebra  
Fall 2007, Vic Reiner  
Midterm exam 2- Due Wednesday November 14, in class  

Instructions: This is an open book, open library, open notes, open web, take-home exam, but you are not allowed to collaborate. The instructor is the only human source you are allowed to consult.

1. (15 points total) Artin’s Chapter 2 Miscellaneous Problems # 3 on p. 77.

2. (15 points total) Artin’s Chapter 2 Miscellaneous Problems # 4 on p. 77.

3. (20 points total; 10 points each part)  
(a) Prove that a group of order 45 must be abelian.  
(b) Exhibit an explicit example of a group of order 45 which is not cyclic, with proof that it is not cyclic.

4. (20 points total; 10 points each part) Artin’s Exercise 3.2.15 on p. 105.

5. (15 points total) Believing that 547 is prime, use Euclid’s algorithm to find the multiplicative inverse $10^{-1}$ of 10 in the finite field $\mathbb{F}_{547}(:= \mathbb{Z}/547\mathbb{Z})$.  
(Using a brute force exhaustive search will earn no credit on this problem, but is fine as a check.)

6. (15 points total) Find a basis over the field $\mathbb{F}_7$ for the subspace  
$\ker A := \{ X \in \mathbb{F}_7^3 : AX = 0 \}$  
of $\mathbb{F}_7^3$, where $A \in \mathbb{F}_7^{2 \times 3}$ is the matrix  
\[
A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}.
\]
Show your work, that is, don’t just write down an answer.