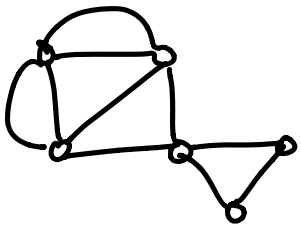
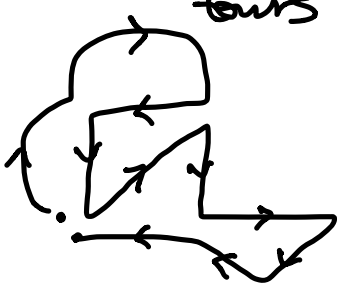
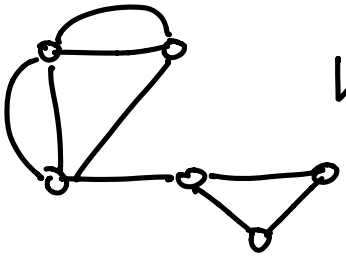
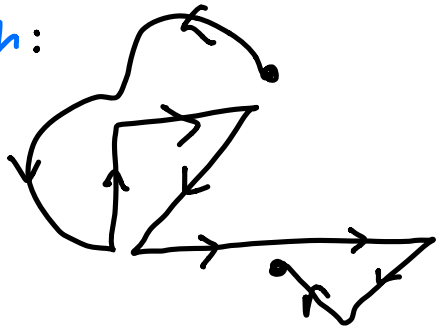
Math 4707 GROUP WORK on Euler paths/walks

Recall that  has (many) Euler tours

e.g. 

while  has no Euler tour, but does have an Euler path:



QUESTIONS

① Try to come up with a conjecture that characterizes graphs $G=(V,E)$ having an Euler path, but no Euler tour, similar in spirit to the one we proved for graphs having an Euler Tour. Can you prove it?

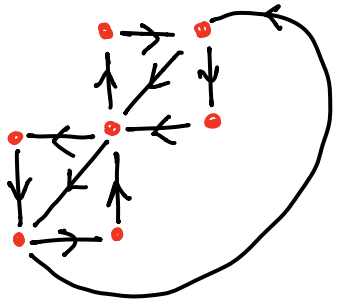
(2) Consider a directed graph (digraph) $D = (V, A)$

V = vertices, A = directed arcs

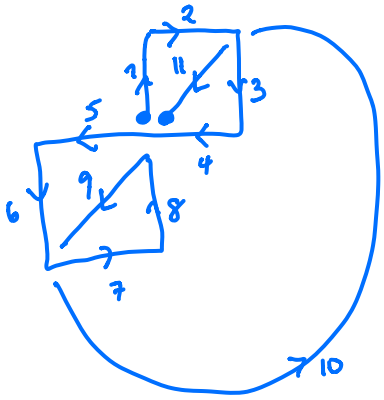
and directed Euler tours

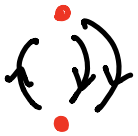
:= sequences of arcs that start at and end at a vertex v_0, follow the arrows along arcs and traverse each arc in A exactly once

e.g.

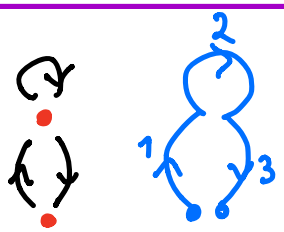


has directed Euler tour

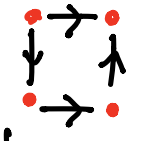




has none



has some



has none

has some

Can you write down a characterization of which digraphs $D = (V, A)$ have directed Euler tours, similar to the undirected case?