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Infinite Toeplitz matrices

\[
\begin{bmatrix}
\cdots & 1 & a_1 & a_2 & a_3 & \cdots \\
1 & a_1 & a_2 & a_3 & \cdots \\
a_1 & a_2 & a_3 & \cdots \\
a_2 & a_3 & \cdots \\
a_3 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]

\(\langle \text{unr} \rangle\) power series \(1 + at + a_2 t^2 + \ldots = a(t)\)

The groups under multiplication correspond.

Consider the subset of (TP) \underline{totally positive} matrices,

i.e. all minors \(\geq 0\)

\[a_1, a_2, \ldots \geq 0\]

\[a_1^2 - a_2, a_2^2 - a_4, a_4^2 - a_6, \ldots \geq 0\]

THM (Edrei-Thuong)

TP matrices \(a(t)\) can be written

\[a(t) = e^{st} \frac{I_T (1 + at)}{I_T (1 - bt)}\]

where \(X, \alpha_i, \beta_j \geq 0\)

\[\alpha_1 \geq \alpha_2 \geq \ldots\]

\[\beta_1 \geq \beta_2 \geq \ldots\]

Now consider infinite symmetric Toeplitz matrix

\[
\begin{bmatrix}
1 & 0 & 0 & \cdots \\
A & -a_1 & -a_2 & a_3 \\
-a_1 & A & -a_1 & -a_2 \\
-a_2 & -a_1 & A & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]

\[A = \sum a_i < \infty\]

with the condition that circular minors are \(\leq 0\)

\[\Delta_{i,j}\]

\[e.g., -a_1 \leq 0\]

\[\Delta_{i,j} = \begin{vmatrix}
a_2 & a_3 \\
a_1 & a_2
\end{vmatrix} = a_2 - a_1 a_3 \leq 0\]

\[\text{(Circular minors)}\]
**Problem:** Find an Edrei-Thoma-style characterization of these matrices.

**Motivation:** Elec. network response matrices

boundary vertices: 1, 2, ...

Put 1 V battery at i and measure current through $j$ = (i, j) entry of response matrix

Building blocks construct new ones via

Schur complement of $X$ in $\begin{bmatrix} X & Y \\ Z & T \end{bmatrix}$ is $K = X - YT^{-1}Z$

Can one answer the problem by making some basic building blocks via Schur complements?