For $G := \text{Out}(F_n) = \text{outer automorphisms of free group } F_n = \langle a_1, \ldots, a_n \rangle$ on $n$ elements

\[ \text{:= } \frac{\text{Aut}(F_n)}{\text{Inn}(F_n)} \]

\[ \text{inner automorphisms } = \{ \text{conjugations by } g \text{ in } G \} \]

\[ x \mapsto gxg^{-1} \]

(Culler & Vogtmann 1986) produced such an

\[ \text{the spine of outer space!} \]

\[ \rightarrow X/\Theta G \text{ stabilizes finite} \]

\[ X/\Theta G = \Sigma \text{ finite} \]

Smillie & Vogtmann 1987 used it to show

\[ X(\text{Out}(F_n)) = \chi_{1,n} + \chi_{2,n} + \cdots + \chi_{2n-2,3n-3} \]

\[ \chi_{v,e} := \frac{1}{v!e!2^e} \sum_{G \text{ admissible}} \tau(G) \]

where $\tau(G) = (-1)^{v-1} \tau_G(0,1)$

\[ \tau(\ast) = +1 \]
\[ \tau(\emptyset) = +1 \]
\[ \tau(\circ) = 0 \]
\[ \tau(\O) = -1 \]
\[ \tau(\Theta) = -2 \]
\[ G \text{ a group } \implies \text{ group cohomology } H^*(G) \]

\[ \text{def in } \]

\[ H^*(BG) \]

where \[ * \approx EG \times G \text{ free, i.e. all cells have trivial stabilizers} \]

\[ \downarrow \]

\[ BG = EG/G \]

If \( BG \) was a finite cell complex,

then \[ \chi(G) \overset{\text{def in}}{=} \sum_{i \geq 0} (-1)^i \text{rank } H^i(BG) \]

\[ = \sum (-1)^{\text{dim}(o)} \sum_{\text{cells } o \text{ in } BG} * \]

Sometimes they can't find such an \( EG \) and \( BG = EG/G \)

\[ \approx X \text{ free with } BG \text{ finite,} \]

but instead they can find \[ * \approx X \text{ with stabilizers of cells finite,} \]

\[ \downarrow \]

\[ X/G = \mathbb{R} \text{ finite} \]

and compute the rational Euler characteristic

\[ \chi(G) := \sum_{o \in \mathbb{R}} \frac{(-1)^{\text{dim}(o)}}{|\text{stab}_G(o)|} \]
In[13] := Series[Chi, {z, 0, 6}]

Out[13] = \[\sum_{n=0}^{6} \frac{z^n}{n!} \]

Out[5] = \{1, 1, 1, 1, \frac{z}{2}, \frac{z^2}{4}, \frac{z^3}{8}, \frac{z^4}{16}, \frac{z^5}{32}, \frac{z^6}{64}\}


Out[6] = \{0.125, -0.166667\}


Out[7] = \{1, 7, 3, 1\} \rightarrow \text{sums to} \ -\frac{1}{48}

In[8] := N[Cantidiag[3]]

Out[8] = \{0.0208333, -0.291667, 0.75, -0.5\}


Out[9] = \{1, 31, 583, 163, 71, 71\} \rightarrow \text{sums to} \ -\frac{1}{384}

In[10] := N[Cantidiag[4]]

Out[10] = \{0.00260417, -0.19375, 2.02431, -6.79167, 8.875, -3.94444\}


Out[11] = \{1, 113, 175, 14473, 1559, 8665, 633, 211\} \rightarrow \text{sums to} \ -\frac{1}{3840}

Out[10] = \{0.00260417, -0.19375, 2.02431, -6.79167, 8.875, -3.94444\}