1. (15 pts.) Find solutions to the following differential equation, describing a harmonic motion:

\[
\frac{d^2 y(t)}{dt^2} + 4y(t) = 0.
\]

2. (15 pts.) An object is released from rest at a height of 100m above the ground. Let \( y(t) \) denote the displacement of this object from its initial position at time \( t \). Then, neglecting frictional forces, this function \( y(t) \) satisfies the initial-value problem:

\[
\frac{d^2 y(t)}{dt^2} = g, \quad y(0) = 0, \quad \frac{dy(t)}{dt}(0) = 0.
\]

Here, \( g \) is a constant, the gravitational constant. Find the time when the object hits the ground. In other words, find the time \( t_{100} \) such that

\[ y(t_{100}) = 100. \]

3. (20 pts.) Use “separation of variables” to find solutions to the differential equation:

\[
(1 + y) \frac{dy(x)}{dx} = x \cos x. \tag{1}
\]

4. (a) (10 pts.)

Solve the quadratic equation,

\[
y^2 + 2y - 2(x \sin x + \cos x) = 1, \tag{2}
\]

for the variable \( y \) in terms of the variable \( x \).

(b) (10 pts.) Show that your function \( y = y(x) \) of part (a) satisfies the differential equation (1) of Problem 3.

5. Let the given function \( y(x) \) of the variable \( x \) satisfy the algebraic equation (2) of Problem 4a. Prove that \( y(x) \) also satisfies the differential equation (1) of Problem 3. Hint: Differentiate both sides of the algebraic equation (2) with respect to \( x \) and use the chain rule.

6. Find solutions to the differential equation:

\[
\frac{dy}{dx} + \frac{1}{x}y = 1 \ln x, \quad y(1) = 3.75.
\]