1. Let,

\[ \mathbf{r}(t) = \mathbf{i} \cos t^2 + \mathbf{k} \sin t^2 + \mathbf{k}, \quad t \geq 0. \]

(a) (15 pts.) Reparametrize this curve, using the arclength.
(b) (15 pts.) Find the curvature of the reparametrized curve at the point \( s = 1 \).

2. (15 pts.) Let \( \mathbf{r}(t) \) be a given differentiable curve on the unit sphere. In other words, assume that this function is differentiable and that \( |\mathbf{r}(t)| = 1 \). Show that at each point \( \mathbf{r}(t) \) of this curve, the tangent vector is perpendicular to this position vector. In other words, show that

\[ \mathbf{r}'(t) \cdot \mathbf{r}(t) = 0. \]

3. (20 pts.) Let \( \mathbf{r}(t) \) be a given differentiable curve such that \( \mathbf{r}'(t) \neq \mathbf{0} \). As usual define the tangent unit vector to this curve by equation:

\[ \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}. \]  \hspace{1cm} (1)

Prove that

\[ |\mathbf{T}'(t)| = \frac{|\mathbf{r}'(t) \wedge \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^2}. \]  \hspace{1cm} (2)

4. (15 pts.) Find the velocity and acceleration of a particle with position function

\[ \mathbf{r}(t) = e^t (\mathbf{i} \cos t + \mathbf{j} \sin t + \mathbf{k} t) \]

5. (15 pts.) Find a point on the curve,

\[ \mathbf{r}(t) = \mathbf{h}^3 + 3 \mathbf{j} t + \mathbf{k} t^2 \],

where the normal plane is parallel to the plane,

\[ 6x + 6y - 8z = 1. \]

6. (15 pts.) The following problem is motivated by Kepler’s Second Law of planetary motion, which says that the areal velocity is constant. Let \( \mathbf{r}(t) \) be a given differentiable curve such that \( \mathbf{r}''(t) \) is parallel to \( \mathbf{r}(t) \).

Show that

\[ (\mathbf{r}'(t) \wedge \mathbf{r}(t))' = 0. \]