1. Give an example of a vector space and three linearly independent vectors in it.

2. (a) Let \( V \) be an abstract vector space and let \( v_1, v_2, \) and \( v_3 \) be vectors in \( V \). Define that these vectors are linearly independent.
   (b) Define that these three vectors span \( V \).
   (c) Define that these three vectors form a basis for \( V \).

3. Let the matrix \( A \) be given by,
   \[
   A = \begin{bmatrix}
   2 & 1 & 1 \\
   4 & -6 & 0 \\
   -2 & 7 & 2 
   \end{bmatrix}.
   \]
   (a) Find the dimension of the space of column vectors of \( A \).
   (b) Find the dimension of the space of raw vectors of \( A \).
   (c) (This part is independent of the matrix \( A \).) Recall that all 3x3 matrices form a vector space with respect to componentwise addition and componentwise multiplication by a scalar. Find the dimension of this vector space.

4. Let \( V \) be a vector space and let \( b_1 \) be a set of basis vectors consisting of the single element \( b_1 \). Next let \( c_1, c_2, \ldots, c_m \) be another set of basis vectors. Prove that \( m = 1 \). Hint: First show that it is no loss of generality to assume that \( m = 2 \).

5. Finally a mathematical pathology from our text. We define a "goofy" addition on \( \mathbb{R}^2 \) by:
   \[
   (a_1, a_2) +_g (b_1, b_2) = (a_1 + b_1 + 1, a_2 + b_2 + 1)
   \]
   and keep the definition of multiplication by a scalar unchanged. Prove that \( \mathbb{R}^2 \) is not a vector space with respect to the addition \( +_g \).

GOOD LUCK