Math 5385 – Homework I
Due Wednesday, February 2

From the text.

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Hints: For problem 1.2.7a you need to show that if \( r = \sin 2\theta \) for some \( \theta \), then \( (x^2 + y^2)^3 = 4x^2y^2 \). For 1.2.7b you need to show that if \( (x^2 + y^2)^3 = 4x^2y^2 \) then \( r = \sin 2\theta \) for some \( \theta \).

For problem 1.4.7, first determine what is \( V(x^n, y^m) \)?

Plus the following additional problems.

1. Which of the following statements about division of integers are true and which are false?

\[
\begin{align*}
1 | 7 & \quad 7 | 1 & \quad 0 | 7 & \quad 7 | 0 & \quad 0 | 0 & \quad 2 | (-6) & \quad (-2) | 6 & \quad 2^8 | 10!
\end{align*}
\]

2. Find \( d = \gcd(1122, 1265) \) “by hand” and express it as an integer combination \( d = k \cdot 1122 + l \cdot 1265 \). Is \( 7 \in \mathcal{I}(1122, 1265) \)?

3. Given two cups holding 16 oz. and 25 oz. and a lake full of water, how can you measure out exactly 3 oz. of water? Be careful not to spill.

4. Given two points in a plane, a Cassini curve is given by setting the product of the distances equal to a constant (whereas an ellipse was given by setting the sum equal to a constant. Take the points to be \((\pm c, 0)\) and let the product \( d_1d_2 = 2a \). Find a polynomial equation \( f(x, y) = 0 \) for the resulting Cassini curve. See if you can make your calculator or computer draw some of these curves for you, for example the cases \( c = 1, a = 1 \) and \( c = 1, a = \frac{1}{2} \).