# Math 5525 – Homework III – revised

(Due Wednesday, March 7)

## From the text.

| Chapter | Problems |
|---|---|
| 5 | 5adf, 6, 14af, 15ad |
| 6 | 1adf, 4,5,9,12abcj, 14 |

**Additional instructions and hints:** For chapter 5, problem 6 just list the canonical forms which are really in different conjugacy classes, i.e., don't consider forms which only differ by reordering the eigenvalues as different.

For problems 14 and 15 try to give a careful argument about why the given set of matrices is open and why it is dense (or why not). You may use the fact that "the eigenvalues depend continuously on the matrix". More precisely, given any matrix $A$ and any $\epsilon > 0$, there is a $\delta > 0$ such that if $B$ is a matrix with $d(A, B) < \delta$ then every eigenvalue $\mu$ of $B$ is within distance $\epsilon$ of some eigenvalue $\lambda$ of $A$. Here $d(A, B)$ is distance in matrix space and the conclusion is that $|\mu - \lambda| < \epsilon$ in the complex plane.

More precisely, suppose $\lambda_1, \dots, \lambda_k$ are the distinct eigenvalues of $A$ with multiplicities $e_1, \dots, e_k$ and let $\epsilon > 0$ be chosen so that the $\epsilon$ neighborhoods of the $\lambda_i$ are disjoint.Then there is a $\delta > 0$ such that every matrix $B$ with $d(A, B) < \delta$ has $e_i$ eigenvalues $\mu$ with $|\mu - \lambda_i| < \epsilon$ (counting any repeated eigenvalues with multiplicities). In the case of distinct eigenvalues for $A$ we have $n$ disjoint neighborhoods and it follows that $B$ has $n$ disjoint eigenvalues, one in each neighborhood.

Another useful fact is that adding a small multiple of the identity $kI$ to a given matrix $A$ just moves all the eigenvalues from $\lambda$ to $\lambda + k$.

For chapter 6, problem 4, express the answer in terms of the initial vector $X_0 = X(0)$, i.e., $X(t) = \exp(tA)X_0$.