Math 5535 – Homework I

Hand in the starred problems only on Wednesday, September 23

From the text.

Pages Problems
23–25 *4, *5, 6, 7, 9, *10 (see hint), 14
34–36 1, 2, *4, *14, *16 (see hint)
41–43 1 (see hint), 4, 8

Hints: Problem 10, page 25. Since \( f(I) \supset I \) there exist points \( c, d \in I \) with \( f(c) = a \) and \( f(d) = b \). Consider the graph of \( f \) over the interval between \( c \) and \( d \). There are several cases corresponding to whether \( c < d \) or \( d < c \) and whether or not \( c \) or \( d \) equal \( a \) or \( b \).

Problem 16, page 35. According to the definition on page 12, you need to prove both stability and attractivity for \( f \) whereas you are given those two things for the second iterate \( f^2 \). Consider the even iterates \( f^{2n}(x) \) and odd iterates \( f^{2n+1} \) separately and use the fact that \( f^{2n+1}(x) = f^{2n}(f(x)) \).

Problem 1, page 41. Use the hint for the additional problem 1 below.

Plus the following additional problems.

1*. Consider the quadratic maps \( f(x) = x^2 - c \) where \( c \) is a constant. Determine for which values of \( c \) there exists an orbit of minimal period two. Find the two points \( x_0, x_1 \) of the orbit and compute the multiplier \( \mu = f'(x_0)f'(x_1) \). For which values of \( c \) is the orbit an attractor (i.e., asymptotically stable)? Hint: the formula below should be useful:

\[
x^4 - 2cx^2 + c^2 - c = (x^2 - x - c)(x^2 + x + 1 - c).
\]

2*. Let \( f(x) = 1 + \tanh(x) = \frac{2}{1 + e^{-2x}} \). Prove that \( x \) has a globally attracting fixed point \( \bar{x} \in [1, 2] \), i.e., \( \bar{x} \) is a fixed point such that for every initial state \( x_0 \in \mathbb{R} \), we have \( x_n \to \bar{x} \) as \( n \to \infty \).