From the text.

Pages Problems
353 5*, 6* (i.e. when \(c\) lies in the given circle)
370 7*

*1. For each of the following complex functions \(F(z), z = x + iy\), express \(F\) as a real mapping of the plane

\[
F(x, y) = \begin{bmatrix} f(x, y) \\ g(x, y) \end{bmatrix}
\]

then compute the complex derivative \(F'(z)\) in two ways, first by differentiating the formula \(F(z)\), second by computing the Jacobian matrix \(DF(x, y)\) and using the correspondence between similarity matrices and complex numbers: \[
\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \rightarrow a + ib. \] (Of course the results of these two computations should agree.)

(a) \(F(z) = z^2 + (1 - i)z\)
(b) \(F(z) = z^3 + z + 1\)
(c) \(F(z) = \frac{1 - z}{1 + z}\)

*2. For the maps \(F(z), G(w)\) below, find a conjugacy \(w = h(z)\) between \(F\) and \(G\) which is a fractional linear (Möbius) transformation:

\[
h(z) = \frac{az + b}{cz + d}
\]

(The coefficients \(a, b, c, d\) are not uniquely determined since multiplying them all by a constant does not change \(h(z)\). So you may as well simplify the computation by choosing one of them to be 1.)

\[
F(z) = \frac{z}{2} \quad G(w) = \frac{1 + 3w}{3 + w}
\]

*3. For each of the following complex maps \(F(z)\), use the transformation \(w = 1/z\) to analyze the stability of the fixed point at infinity, i.e., first find the conjugate map \(G(w)\) and then analyze its fixed point at \(w = 0\).

(a) \(F(z) = 2z - 3\)
(b) \(F(z) = z^2 + i\)
(c) \(F(z) = \frac{z^2 + 1}{2z}\)

*4. Show that when \(c = -i\), the Julia sets \(K_c\) and \(J_c\) for the quadratic map \(F(z) = z^2 + c = z^2 - i\) are connected. Also show that \(F(z) = z^2 - i\) has no attracting periodic orbits. What about when \(c = 1 - i\)? Hint: Look at the critical orbit.