Math 5583 – Homework I
Due Monday September 19

From the text.
Chapter Problems
1 2ii, 3, 4ii,iii,iv, 5, 9, 10, 12, 13, 14

Additional problems.
1. a. Consider the transformation of the plane obtained by first translating by the vector (1, 2) then scaling by a factor of 3, then rotating clockwise by $\pi/3$ radians. Express this transformation as a complex affine function $f(z) = cz + d$ for certain $c, d \in \mathbb{C}$.
b. Let $f(z)$ be the composition of any number of translations, rotations and scalings of the plane (composed in any order). Show that $f(z) = cz + d$ for some $c, d \in \mathbb{C}$.
2. The polar curve $r = \cos \theta, -\pi/2 < \theta < \pi/2$, is a circle passing through the origin, tangent to the $y$-axis. Show that the image of this circle (in the $z$-plane) under the map $w = z^2$ is the cardioid (in the $w$-plane) with polar equation $R = \frac{1}{2}(1 + \cos \Theta)$. Draw a picture.