First of all

1. Solve all the recommended exercises from sections 14.7 to 15.5.

2. Study and solve the problems in all the quizzes.

3. Take a look at the webpages of the other sections and solve their problems too. The previous exams that Bob has in his page are good enough for practice. The links are here

- Rafaek’s: http://www-users.math.umn.edu/~rmonteir/m2263.html
- Bob’s: http://www.math.umn.edu/~gulliver/2263/
- Antoine’s: http://www.math.umn.edu/~apauthie/contenu_du_site/math2263.html

The following problems are based on exercises of [1, Chapter 11].

Problem 2 - [1, Chapter 11]
Interchange the order of integration

1. \( \int_1^e \left[ \int_0^{\log(x)} f(x,y) \, dy \right] \, dx \),

2. \( \int_{-1}^1 \left[ \int_{-\frac{1-x^2}{\sqrt{1-x^2}}}^1 f(x,y) \, dy \right] \, dx \),

3. \( \int_{\pi}^{e} \left[ \int_{\sin(\frac{x}{2})}^{\sin(x)} f(x,y) \, dy \right] \, dx \),

4. \( \int_{0}^{1} \left[ \int_{x^3}^{x^2} f(x,y) \, dy \right] \, dx \),

Problem 3 - [1, Chapter 11]
Let \( A = \int_0^1 e^{-t^2} \, dt \) and \( B = \int_0^{1/2} e^{-t^2} \, dt \). Evaluate the iterated integral

\[
I = 2 \int_{-1/2}^1 \left[ \int_0^x e^{-y^2} \, dy \right] \, dx
\]
in terms of \( A \) and \( B \), showing that there are positive integers \( m \) and \( n \) such that

\[
I = mA - nB + e^{-1} - e^{-1/4}.
\]
Problem 5

Parametrize the domain $E$ in two ways

$$E = \left\{ (x, y) \in \bigcup_{i=1}^{N} \mathbb{R}^2 | a_i \leq x \leq b_i, l_i(x) \leq y \leq u_i(x) \right\} = \bigcup_{j=1}^{M} \left\{ (x_j, y_j) \in \mathbb{R}^2 | a_j \leq y \leq b_j, \tilde{l}_j(y) \leq y \leq \tilde{u}_j(x) \right\}$$

when

a) $E$ is the bounded region given by the intersection of the curve $y = x - x^2$ and $y = x - 1$.

b) $E$ is the bounded region given by the intersection of the curve $y = 2 - x^2$ and $y = x^2$.

Problem 6

Integrate

$$\int_{x=0}^{1} \left[ \int_{y=\sqrt{x}}^{1} \sqrt{1+y^3} dy \right] dx$$

Problem 7

Find the area of the bounded region outside the circle $x^2 + y^2 = 1$ and inside the circle $x^2 + y^2 = 2x$.

References