Please answer the following questions completely and clearly. An unsupported answer is worth few points.

(1) Are the provided functions solutions to the differential equations? Justify your answers.
   (a) \( y' + 4y = 0, \ y_1 = \cos(2x), \ y_2 = \sin(2x) \)
   We compute:
   \[ y_1' = -2\sin(2x), \text{ then } y' + 4y = -2\sin(2x) + 4\cos(2x) \neq 0. \]
   \[ y_2' = 2\cos(2x), \text{ then } y' + 4y = 2\cos(2x) + 4\sin(2x) \neq 0. \]
   Thus, neither \( y_1 \) or \( y_2 \) are solutions.

   (b) \( x^2y'' + xy' - 4y = 4\ln(x), \ y_1 = x^2 - \ln(x), \ y_2 = x - \ln(x) \)
   \[ y_1' = 2x - \frac{1}{x}, \ y_1'' = 2 + \frac{1}{x^2}. \]
   Then,
   \[ x^2y'' + xy' - 4y = 2x^2 + 1 + 2x^2 - 1 - 4x^2 + 4\ln(x) = 4\ln(x) \]
   \[ y_2' = 1 - \frac{1}{x}, \ y_2'' = \frac{1}{x^2}. \]
   Then,
   \[ x^2y'' + xy' - 4y = 1 + x - 1 - 4x + 4\ln(x) = 4\ln(x) - 3x \neq 4\ln(x). \]
   Thus, \( y_1 \) is a solution but \( y_2 \) is not.

(2) Solve the following IVP

\[
\begin{align*}
\frac{dy}{dx} &= x\cos(x^2) \\
y(\sqrt{\frac{\pi}{2}}) &= 5
\end{align*}
\]

We integrate

\[
\int x\cos(x^2)dx = \int \frac{1}{2} \cos(u)du = \frac{1}{2} \sin(x^2) + C
\]

Using \( y(\sqrt{\frac{\pi}{2}}) = 5 \), we get \( C = \frac{9}{2} \). So,
\[ y = \frac{1}{2}(\sin(x^2) + 9) \]

(3) A baseball is thrown straight downward with an initial speed of 40 ft/s from the top of the Washington monument (555 ft tall). How long does it take to reach the ground, and with what speed does the baseball strike the ground?

We have that \( \frac{dv}{dt} = -32 \) and \( v(0) = -40 \). This gives that \( v(t) = -32t - 40 \).

This gives an IVP in position: \( \frac{dx}{dt} = -32t - 40 \) and \( x(0) = 555 \). Integrating gives \( x(t) = -16t^2 - 40t + 555 \).

We are interested in how long it takes to reach the ground and how fast the ball is going. We solve for some \( T \) when \( x(T) = 0 \). This means \(-16t^2 - 40t + 555 = 0 \). Using the quadratic equation we find:
\[
T = \frac{-40 \pm \sqrt{1600 + 16 \cdot 4 \cdot 555}}{-32}
\]

Taking the positive root, we have \( T = 4.77 \)s and get \( v(4.77) = -192.66 \)