Please answer the following questions completely and clearly. An unsupported answer is worth few points.

(1) Find the equation of the sphere which contains the two antipodal points \( P_1(3, 7, 1) \) and \( P_2(2, 4, 6) \).

We begin by finding the radius. Since the two points are antipodal that means that \(|P_1P_2|\) is the diameter. We compute:

\[
|P_1P_2| = \sqrt{(3 - 2)^2 + (7 - 4)^2 + (1 - 6)^2} = \sqrt{1 + 9 + 25} = \sqrt{35}
\]

So our radius is half that.

The center of our circle is the midpoint of our two points so it is \( C\left(\frac{5}{2}, \frac{11}{2}, \frac{7}{2}\right) \). The sphere’s equation is:

\[
(x - \frac{5}{2})^2 + (y - \frac{11}{2})^2 + (z - \frac{7}{2})^2 = \frac{35}{4}
\]

(2) Given vectors \( \vec{u} = \langle 1, 4, 8 \rangle \) and \( \vec{v} = \langle 3, 2, 1 \rangle \) find

(a) The unit vector in the direction \( \vec{v} - \vec{u} \)

We first compute the vector \( \vec{w} = \vec{v} - \vec{u} \)

\( \vec{w} = (3 - 1, 2 - 4, 1 - 8) = \langle 2, -2, -7 \rangle \) and \( |\vec{w}| = \sqrt{4 + 4 + 7} = \sqrt{57} \)

So a unit vector in the direction of \( \vec{v} - \vec{u} \) is \( \left\langle \frac{2}{\sqrt{57}}, \frac{-2}{\sqrt{57}}, \frac{-7}{\sqrt{57}} \right\rangle \)

(b) \( \text{proj}_\vec{u}\vec{v} \)

We know from class that

\[
\text{proj}_\vec{u}\vec{v} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2}\vec{u}
\]

We find that \( \vec{u} \cdot \vec{v} = 3 + 8 + 8 = 19 \) and \( |\vec{u}| = 9 \). So,

\[
\text{proj}_\vec{u}\vec{v} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2}\vec{u} = \frac{19}{81} \langle 1, 4, 8 \rangle
\]

(c) An explicit expression for the angle between \( \vec{u} \) and \( \vec{v} \)

We know that \( \vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta \), where \( \theta \) is the angle in between our two vectors. Rearranging, we have that

\[
\theta = \arccos \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}
\]

With, \( |\vec{v}| = \sqrt{14} \), we have \( \theta = \arccos \frac{19}{9\sqrt{14}} \)