(1) Find and sketch the domain of $f(x, y) = \sqrt{1-x} + \sqrt{4-x^2-y^2}$:

We need that both of the square roots are defined. So we need

i.) $1-x \geq 0$, i.e. $x \leq 1$

ii.) $4-x^2-y^2 \geq 0$ i.e. $x^2+y^2 \leq 4$.

So our domain is all $(x, y)$ in the ball of radius 2 with $x \leq 1$.

(2) Show that the following limit does not exist:

$$\lim_{(x,y) \to (0,0)} \frac{x^5-6y}{y+3x}$$

Along the $x$-axis we have $y = 0$.

So, $\lim_{(x,0) \to (0,0)} \frac{x^5-6y}{y+3x} = \lim_{(x,0) \to (0,0)} \frac{x^5}{3x} = \lim_{(x,0) \to (0,0)} \frac{1}{3} x^4 = 0$.

Along the $y$-axis we have $x = 0$.

Then, $\lim_{(0,y) \to (0,0)} \frac{x^5-6y}{y+3x} = \lim_{(0,y) \to (0,0)} \frac{-6y}{y} = \lim_{(0,y) \to (0,0)} -6 = -6$

(3) Calculate the partial derivatives of $f(x, y) = x^2y \cos(xy)$:

We find that:

$f_x(x, y) = 2xy \cos(xy) - x^2y^2 \sin(xy)$

and

$f_y(x, y) = x^2 \cos(xy) - x^3y \sin(xy)$