Please answer the following questions completely and clearly. An unsupported answer is worth few points.

(1) Using the change of variables \( x = \frac{1}{4} u, y = 5v \) evaluate the integral

\[
\iint_E x \, dA
\]

Where the region \( E \) is the ellipse \( x^2 + \frac{y^2}{100} = 1 \).

We calculate the Jacobian is:

\[
|J(x, y)| = \begin{vmatrix} \frac{1}{4} & 0 \\ 0 & 5 \end{vmatrix} = \frac{5}{4}
\]

Using the substitution above we get \( x^2 + \frac{y^2}{100} = \left( \frac{u}{4} \right)^2 + \left( \frac{5v}{100} \right)^2 = \frac{u^2}{16} + \frac{25v^2}{100} = \frac{u^2}{16} + \frac{v^2}{4} = 1 \). Note: The writer of this quiz made a mistake and accidentally made you make another ellipse. Let’s call this new region \( D = \{(u, v) | \frac{u^2}{16} + \frac{v^2}{4} \leq 1\} \).

Our integral becomes:

\[
\iint_E x \, dA = \iint_D \frac{u}{4} \cdot \frac{5}{4} \, dA
\]

To integrate this, we need only note that we are integrating an odd function over a symmetric domain about the origin. Therefore, the answer is zero.

(2) Find the gradient vector field for \( f(x, y) = x^2(y + 1) + xy \). Plot the vector field at the points \((1, 0), (0, 1), (-1, 0), (0, -1) \text{ and } (1, -1)\).

We calculate that our gradient field is \( F(x, y) = \nabla f(x, y) = \langle 2x(y + 1) + y, x^2 + x \rangle \).

Using this we calculate that the vectors at the above points are:

\[
F(1, 0) = \langle 2, 2 \rangle \\
F(0, 1) = \langle 1, 0 \rangle \\
F(-1, 0) = \langle -2, 0 \rangle \\
F(0, -1) = \langle -1, 0 \rangle \\
F(1, -1) = \langle 0, 2 \rangle
\]