Please answer the following questions completely and clearly. An unsupported answer is worth few points.

(1) Calculate the integral \( \iint_S xyz \, dS \) where \( S \) is given by \( z = 2 + 3x - 4y \) where \( x \in [0, 1] \) and \( y \in [0, 2] \).

We use the formula

\[
|r_x \times r_y| = \sqrt{1 + \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2} = \sqrt{1 + 9 + 16} = \sqrt{26}.
\]

Then

\[
\iint_S xyz \, dS = 2 \int_0^1 \int_0^1 xy(2 + 3x - 4y) \cdot \sqrt{26} \, dx \, dy
\]

\[
= \sqrt{26} \int_0^2 yx^2 + x^3y - 2x^2y^2 \bigg|_0^1 dy = \sqrt{26} \int_0^2 2y - 2y^2 \, dy = \sqrt{26} \left( 2y^2 - 2y^3 \right) \bigg|_0^1 = -\frac{4}{3} \sqrt{26}
\]

(2) Use Stokes’ Theorem to evaluate \( \iint_S \text{curl} \vec{F} \cdot d\vec{S} \), where \( \vec{F} = \langle xy, z, xz \rangle \) and \( S \) is given by \( y = x^2 + z^2 \) and \( y \leq 9 \) and oriented towards the positive \( y \)-axis.

Using Stokes’ Theorem, instead we calculate \( \int \vec{F} \cdot d\vec{r} \). For us, \( \partial S \) is the circle \( x^2 + z^2 = 9 \) and \( y = 9 \). Since we are oriented towards the positive \( y \)-axis we see that we must traverse our circle in a counterclockwise direction. We write \( \vec{r}(t) = \langle 3 \cos t, 9, -3 \sin t \rangle \) for \( t \in [0, 2\pi] \). Then \( \vec{r}'(t) = \langle -3 \sin t, 0, -3 \cos t \rangle \) and \( \vec{F}(\vec{r}(t)) = \langle 27 \cos t, -3 \sin t, -9 \sin t \cos t \rangle \).

Finally, we calculate that

\[
\iint_S \text{curl} \vec{F} \cdot d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) \, dt = \int_0^{2\pi} -81 \sin t \cos t + 27 \sin t \cos t \, dt
\]

\[
= \frac{81}{2} \cos^2 t - 9 \cos^3 t \bigg|_0^{2\pi} = 0
\]