Please answer the following questions completely and clearly. An unsupported answer is worth few points.

(1) Sketch the graph of the function \( f(x, y) = 1 + 2x^2 + 2y^2 \). 

We need to look at the level curves of the function in order to be able to draw a sketch of the graph. Notice that we need \( k \geq 1 \). For \( k = 1 \), we must have \( x = 0 \) and \( y = 0 \), and we have just a point. For \( k > 1 \) we have a family of circles which have larger and larger radius as \( k \) increases. From this we can conclude that our graph must be a circular paraboloid with vertex at \((0, 0, 1)\).

(2) Find \( \lim_{(x,y) \to (0,0)} \frac{x^2ye^y}{x^4+4y^2} \), if it exists, or show that the limit does not exist.

We start by seeing that \( f(x, y) = \frac{x^2ye^y}{x^4+4y^2} \) is undefined for \((0,0)\) and this is exactly the point we are approaching in our limit. We may have a limit which does not exist, but we would have to prove this. Consider approaching \((0,0)\) from two different paths:

1. First let’s approach along the x-axis. For \( x \neq 0 \), \( f(x,0) = 0 \) and so \( f(x,y) \to 0 \) as \((x,y) \to (0,0)\)

2. I might try approaching along the y-axis next, but this would give the same limit. Let’s approach along the path \( y = x^2 \). Then \( f(x,x^2) = \frac{x^2e^{x^2}}{x^4+4x^4} \) for \( x \neq 0 \) and so along this path, \( f(x,y) \to 1/5 \) as \((x,y) \to (0,0)\)

Since the two limits are not equal, we have shown that the limit does not exist.