Please answer the following questions completely and clearly. An unsupported answer is worth few points.

(1) Evaluate the double integral \( \int \int (x^2 + 2y) \, dA \) where \( D \) is bounded by \( y = x, y = x^3, \) AND \( x \geq 0. \)

Setting the two curves equal, we find:

\[
x = x^3 \implies x^3 - x = 0 \implies x(x^2 - 1) = 0 \implies x = 0, 1, -1
\]

The problem asks for \( x \geq 0 \) and so the x-bounds of our integral are \( 0 \leq x \leq 1. \) By a quick sketch, or by plugging in a test point between 0 and 1, we see that our y bounds are \( x^3 \leq y \leq x, \) and so our integral becomes:

\[
\int_0^1 \int_{x^3}^{x} (x^2 + 2y) \, dy \, dx
\]

Integrating:

\[
= \left[ \frac{x^3 y + y^2}{2} \right]_{x^3}^{x} = \left[ x + x^2 - x^5 - x^6 \right]_0^1 = \left( \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{6}x^6 - \frac{1}{7}x^7 \right)_0^1 = \frac{23}{84}
\]

Note: we could have evaluate this integral using \( dxdy \) as well.

(2) Find the volume of the solid bounded by the cylinder \( x^2 + y^2 = 1 \) and the planes \( y = z, x = 0 \) and \( z = 0 \) in the first octant.

The region we are integrating over is given by the 1/4 circle of radius 1 in the xy-plane. Depending on how you choose your bounds, we can have two different integrals:

\[
\int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx \quad \text{or} \quad \int_0^1 \int_0^{\sqrt{1-y^2}} y \, dx \, dy
\]

The integration will be analogous. Let's integrate the first:

\[
\int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx = \left[ \frac{y^2}{2} \right]_0^{\sqrt{1-x^2}} - x^2 \left[ \frac{1}{2} \right]_0^1 = \left( \frac{1}{2}(x - \frac{1}{3}x^3) \right)_0^1 = \frac{1}{3}
\]