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1. Matching: d; i; a; g; f; l; e; m.
2. True/False: F; T; F. The first of these turned out to be trickier than I intended. The Majority Criterion says that a candidate who has more than 50% of the vote should win the election. That is not the same thing as, “a candidate *must* have over 50% of the votes to win. In fact, Minnesota has elected Jesse Ventura (and Tim Pawlenty) as Governor, using the Plurality method (which satisfies the Majority Criterion), and neither of them had a majority of the votes.
3. There are 20 voters total, so a majority – *more* than 50% – is 11 votes, not 10.
- (a) V wins with the Plurality Method.
 (b) H wins with the Borda Count Method. The ranking is H, C, V.
 (c) C wins with the Plurality-with-Elimination Method.
 (d) H wins with the Method of Pairwise Comparisons. The ranking is H, C, V.
 (e) H is the Condorcet Candidate; H wins all of his head-to-head comparisons.
 (f) C wins with the Approval Method.
4. (a) A unanimous vote is required to pass a motion in this system, so all five players have equal power. Hence the last player has $1/5$ or 20% of the power.
- (b) Here are the winning coalitions for this weighted voting system:
 $\{P_1, P_2\}$ — P_1 and P_2 are critical.
 $\{P_1, P_3\}$ — P_1 and P_3 are critical.
 $\{P_1, P_2, P_3\}$ — P_1 is critical.
- So the total number of times all players are critical is 5. Player 1 is critical 3 times, so P_1 's BPI is $3/5$ or 60%. The other two players each have a BPI of $1/5$ or 20%.
- (c) Player 4 can never be critical or pivotal, which means Player 4 must have a BPI and SSPI of 0. In other words, Player 4 has no power, i.e. is a dummy.
- (d)
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| $\langle P_1, P_2, P_3 \rangle$ | $\langle P_2, P_3, P_1 \rangle$ | $\langle P_3, P_1, P_2 \rangle$ |
| $\langle P_1, P_3, P_2 \rangle$ | $\langle P_2, P_1, P_3 \rangle$ | $\langle P_3, P_2, P_1 \rangle$ |
- (e) Including the mayor, there are six voters total. The mayor can only be pivotal in the fifth position – $\langle C, C, C, C, M, C \rangle$. If the mayor comes before the fifth position, there aren't enough council members voting with her to pass a motion. If the mayor is in the sixth position, then the five council members already have enough votes to pass the motion.

So the mayor is only pivotal in the fifth position, and the mayor is in that position $1/6$ of the time. That means the mayor's SSPI is $1/6$ or about 16.67%. (If that last part is confusing, look at the example in your book or talk to one of us.)

5. (a) With four players, a fair share is 25%. So C_1 will bid on anything worth at least 25% to him. That would be s_1 and s_2 . There are a few different answers possible for the second part of the question. One possible answer is:

C_1 gets s_1

C_2 gets s_2

C_3 gets s_3

D gets s_4

(b) Initially, B gets 1 and 2; A gets 5, 6, 7; C gets 9, 10. That leaves 3, 4, and 8 left over.

(c) First, you should use the spaces I left for you to total up each of the columns and figure out the fair shares. A , B , and D all have a fair share of \$4. C 's fair share is \$5.

Based on the bids:

A gets the bear and has her fair share.

B gets the book and receives \$1 from the pot.

C gets the CD and has to pay \$11 into the pot.

D gets none of the objects, but \$4 from the pot.

If you made it this far, you would get a lot of the points, but you're not quite done. If you compare payments in and out of the pot, there's a surplus of \$6. So each child gets an additional \$1.50 from the pot. ($\frac{6}{4} = 1.50$).