Solutions to Exam 1

These are not meant to be detailed solutions. If you can’t figure out why a given answer is correct, talk to me or your TA.

1. i) b.

\[
\frac{\tan(\theta)}{\sec(\theta)} + \frac{1}{\csc(\theta)} = \tan(\theta) \frac{1}{\cos(\theta)} + \frac{1}{\sin(\theta)} = \frac{\sin(\theta)}{\cos(\theta)} \cos(\theta) + \sin(\theta) = \sin(\theta) + \sin(\theta)
\]

ii) c.

iii) d. \(\sin^{-1}(\sin(3\pi/4)) = \sin^{-1}(1/\sqrt{2}) = \pi/4\).

iv) d.

v) a.

2. a) \(\sin^2 30^\circ + \frac{1}{\sec^2 390^\circ} = \sin^2 30^\circ + \cos^2 390^\circ = \sin^2 30^\circ + \cos^2 30^\circ = 1\)

b) \(\tan \frac{\pi}{4} + \cos 2\frac{\pi}{3} = 1 - \frac{1}{2} = \frac{1}{2}\) (Draw a 45-45-90 and 30-60-90 triangle. Note that a point corresponding to the angle \(2\pi/3\) was given to you in 1(ii).

c) \(\csc \frac{\pi}{4} = \sqrt{2}\) – use the same triangle as you used in part (b).

3. \(\cos \theta < 0\) and \(\tan \theta > 0\) means \(\theta\) must be in quadrant three. So to solve this problem you should draw the correct 3-4-5 triangle in the third quadrant; \(\cos \theta = -\frac{3}{5} = \frac{x}{r}\) implies that \(r = 5\), so \(x = -3\). In quadrant three, \(y < 0\) so \(y = -4\). Now you can write down the trig functions using the definitions:

\[
\sin \theta = y/r = -4/5, \quad \csc \theta = r/y = -5/4 \\
\cos \theta = x/4 = -3/5, \quad \sec \theta = r/x = -5/3 \\
\tan \theta = y/x = 4/3, \quad \cot \theta = x/y = 4/3
\]

4. Given \(y = -5 \sin \left(\frac{1}{2}x - \frac{\pi}{2}\right)\), we see that Amplitude = \(|-5| = 5\), Period = \(T = \frac{2\pi}{1/2} = 4\pi\), and the Phase Shift is \(\frac{\pi/2}{1/2} = \pi\). If you’re not sure how to graph a sin curve with this information, ask me or your TA. (Note that we have a negative sign out front, so the curve will go down first.)