The Graphs of Tangent, Cotangent, Cosecant, and Secant

We're going to find the graphs of these function using the same method we used for sin(x) and cos(x). We'll use a table of values to plot some of the points, and then "fill in" the rest of the graph. It will be a little more complicated than before, because these functions aren't continuous everywhere; what this means is that there will be some "breaks" in the graphs – each of them will have vertical asymptotes. More on that later.

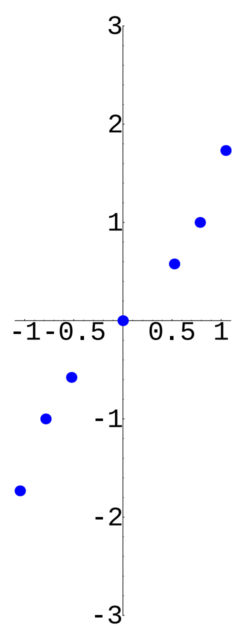
The Graph of Tan(x)

We'll start with the table of values. I'll remind you again: you don't have to memorize these values; you can find all of them using our unit-circle definitions and by fitting a 45-45-90 or 30-60-90 triangle into the circle. We did this during the lecture on section 5.2.

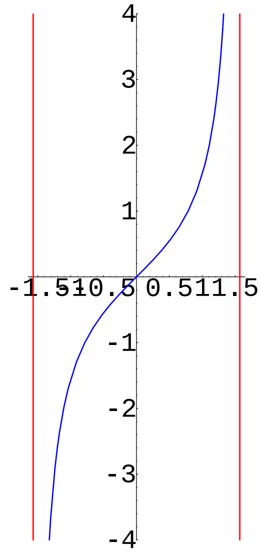
Out[0]=

x	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y = tan(x)	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

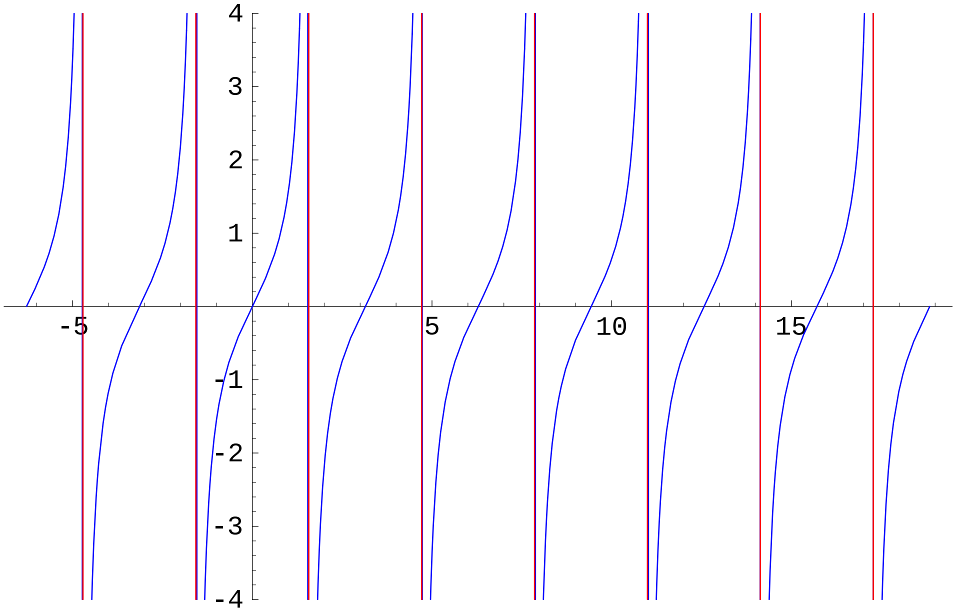
If we plot these points (x, y) they look like this:



We can connect the dots using a smooth curve to get an idea of what the graph looks like, but that's not the whole story yet. As you should recall, $\tan(x) = \frac{\sin(x)}{\cos(x)}$, and therefore tan($x$) is undefined whenever cos(x) = 0. In particular, the tangent function is undefined for $x = \pm\frac{\pi}{2}$. What this means is that we have *vertical asymptotes* at $x = \pm\frac{\pi}{2}$, so the graph extends infinitely down to the left and infinitely high to the right. (Of course, on our graphs we'll have to cut this off at some point.) The red lines here will indicate the asymptotes.



We know that tan(x) is periodic with period π. That means the graph just repeats forever and ever to the left and right.



The Graph of Cot(x)

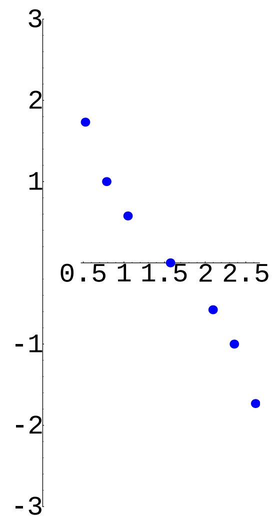
Now we'll do the same thing with $\cot(x)$. The only real difference in our method here is that $\cot(x) = \frac{\cos(x)}{\sin(x)}$ is undefined when sin(x)=0, NOT when cos(x)=0. That means the vertical asymptotes will be in a different place.

First, a table of values:

Out[0]=

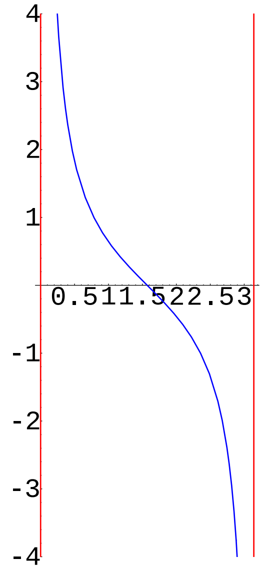
x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
y = cot(x)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$

If we plot these points (x, y) they look like this:

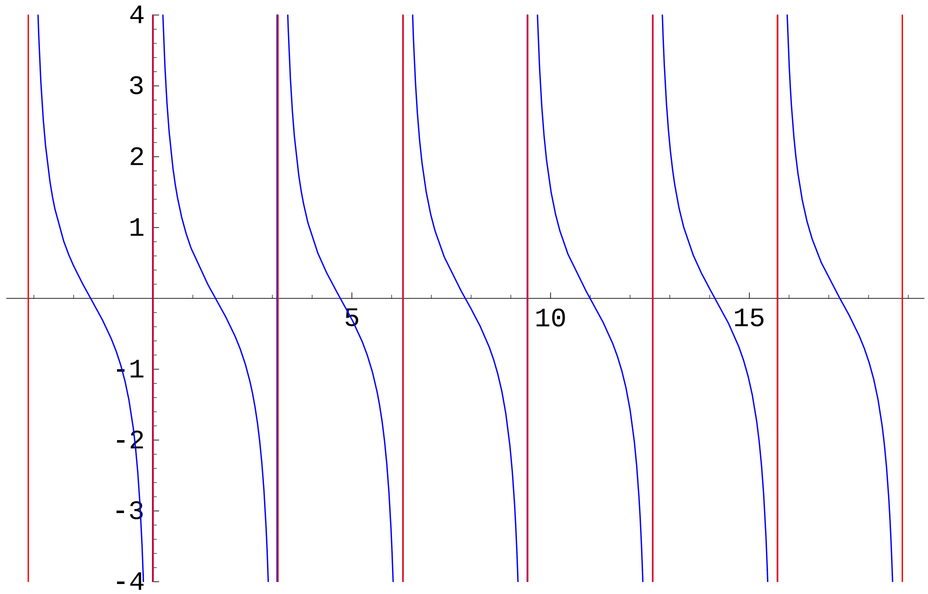


We can connect the dots using a smooth curve to get an idea of what the graph looks like, but that’s not the whole story yet.

As with tan(x), we have to recognize that there are vertical asymptotes to the left and the right of this graph, where $\sin(x) = 0$. (Remember, $\sin(x) = 0$ for multiples of π.)



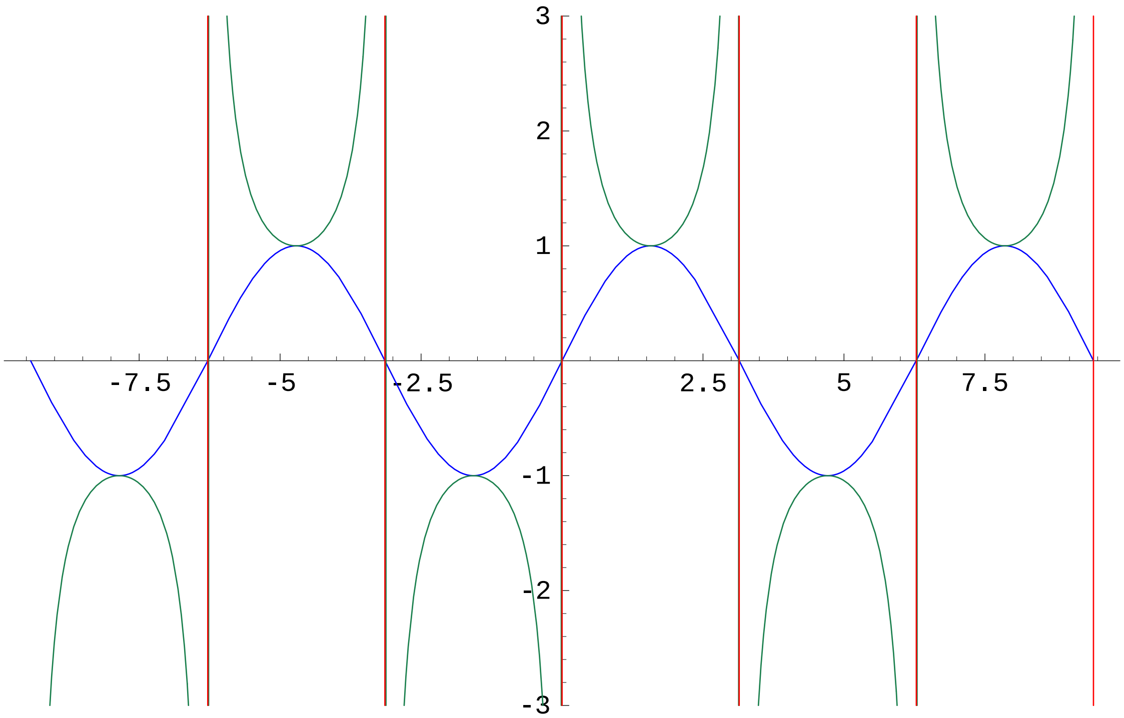
We know that cot(x) is periodic with period π. That means the graph just repeats forever and ever to the left and right.



The Graphs of Csc(x) and Sec(x)

The book doesn't make a particularly big deal about the graphs of csc(x) and sec(x), and we probably won't either. But you should at least see the graphs. As the book points out, if you know what the values of sin(x) and cos(x) are, you can figure out point-by-point what the values of csc(x) and sec(x) are, because you know $\csc(x) = \frac{1}{\sin(x)}$ and $\sec(x) = \frac{1}{\cos(x)}$. Let's examine the cosecant function first.

The first thing you should notice is that csc(x) is *undefined* whenever sin(x)=0, because then $\csc(x) = \frac{1}{0}$, and we can't divide by zero. So it should be no surprise that the graph of csc(x) will have vertical asymptotes at those places where sin(x)=0. Also, csc(x) is positive whenever sin(x) is positive, and csc(x) is negative whenever sin(x) is negative. However, if sin(x) is very small, csc(x) is very *large*, because if you divide 1 by a very small number, you get a large one. (Think about it; $\frac{1}{0.001} = 1000$.) Look at the graph and see if you can see all of these things. The red lines are the asyptotes. The blue line is the graph of sin(x), and the green line is the graph of csc(x).



Now let's look quickly at $\sec(x) = \frac{1}{\cos(x)}$. The vertical asymptotes will now be where cos(x)=0, but the other parts of the graph will look essentially the same – this shouldn't be a surprise, and here's why: the graphs of sin(x) and cos(x) are essentially the same, except for a horizontal shift. Hence the graphs of csc(x) and sec(x) – the *reciprocals* of sin(x) and cos(x) – will look the same, except for a horizontal shift.

Here's the graph. Again, the red lines are the asymptotes, the blue line is cos(x), and the green line is sec(x).