Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. Writing problems should have complete explanations of your work, written in complete sentences with correct grammar.

Due: Thursday, 11/3

Homework Assignment

Regular Problems:
(1) Prove the following sequences diverge according to the definition in Section 16:
   (a) \( a_n = n^2 \)
   (b) \( b_n = \sin \left( \frac{\pi n}{2} \right) \)

(2) For each of the following, prove or give a counterexample.
   (a) If \( s_n \to s \), then \( s \) is an accumulation point of the set of numbers \( \{ s_n \mid n \in \mathbb{N} \} \).
   (b) If \( s \) is an accumulation point of the set of numbers \( \{ s_n \mid n \in \mathbb{N} \} \), then \( s_n \to s \).

(3) For each of the following, prove or give a counterexample. You may use Theorem 17.1 in any proofs, but make sure it applies in the way you use it.
   (a) If \( (s_n) \) converges and \( (t_n) \) diverges, then \( (s_n t_n) \) must be divergent.
   (b) If \( (s_n) \) and \( (s_n/t_n) \) are convergent sequences, then \( (t_n) \) must converge.
   (c) If \( (s_n/t_n) \) converges, then \( t_n \) cannot converge to 0.

(4) Use Theorem 17.1 to find the following limits. Justify each step.
   (a) \( \lim \frac{(n + 1)^2}{n^3 - 5n^2 + 1} \)
   (b) \( \lim \frac{9n + 1}{6 - n} \)

Writing Problem 1: Use Theorem 16.8 to prove that \( s_n = \frac{3n^2 - 1}{2n^3 - 5n} \to 0 \).

Writing Problem 2: Suppose \( (s_n) \) converges to \( s \neq 0 \) and \( (s_n t_n) \) converges to \( L \). Prove that \( (t_n) \) converges. (Hint/Warning: you cannot assume \( s_n \neq 0 \) for all \( n \).)