Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. Writing problems should have complete explanations of your work, written in complete sentences with correct grammar.

Due: Thursday, 11/17

**Homework Assignment**

(As a reminder, since we took a break from homework while preparing for the second exam: this homework covers Section 18 and the beginning of Section 32.)

**Regular Problems:**

1. Prove: \( s_n = ((-1)^n n) \) is an unbounded sequence which does not diverge to \(+\infty\) or \(-\infty\). (Hint: there are three things to prove; make sure you explain why each of them is true.)

2. Use the Monotone Convergence Theorem to show the following sequences converge.
   - (a) \( a_n = \frac{n^2 - 1}{n(n + 1)} \)
   - (b) \( b_n = e^{-n} \). (You may use your prior knowledge of the function \( f(x) = e^x \).)

3. Prove the sequence \( c_n = \frac{1}{n^2} \) is a Cauchy sequence.

4. For each series \( \sum a_n \), find an expression for the partial sum \( s_n = a_1 + a_2 + \cdots + a_n \). Then find the sum of the series or show it is divergent. (In each case you are expected to show supporting work for your answer.)
   - (a) \( \sum_{n=1}^{\infty} \frac{3}{(3n + 2)(3n - 1)} \)
   - (b) \( \sum_{n=1}^{\infty} \frac{(n - 1)!}{(n + 1)!} \)

**Writing Problem 1:** Prove that the following sequence converges and find its limit.

\[
b_1 = 1, \quad b_{n+1} = \sqrt{12 + b_n}
\]

**Writing Problem 2:**

- (a) Use induction to prove \( 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r} \) for \( r \neq 1 \).
- (b) For \( |r| < 1 \), prove carefully (using the sequence of partial sums and the limit laws in Section 17) that

\[
\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r}
\]