

The third exam picks up where the second left off, covering Sections 16, 17, 18, and 32. That doesn't sound like much compared to the 6-8 sections covered on each of the previous midterms, but these sections are very dense and have a lot of material. Don't get caught thinking "I learned about sequences and series in my Calculus course, so I don't need to study as much for this test."

To study for this test you should learn the definitions and theorems, but **there is no substitute for doing problems**. You might be able to look at a sequence and determine its limit using methods from Calculus, but if you haven't worked out limits using the precise methods of Section 17, you will be hard pressed to do so on the test. You might be able to write down the definition of $s_n \rightarrow s$ from Section 16, but if it's been three weeks since you used that definition to prove a certain sequence converges, it will be hard on the exam. If you had a high level calculus course, you may have learned the Monotone Convergence Theorem in a previous class – but in this class you need to be able to both use it and prove that it's true.

If you're nervous about proving things or using $\epsilon - N$ definitions on the exam, let me state again that we are not looking to surprise you with any of the problems on the test. Lectures, homework assignments and writing quizzes give you a good idea of what I feel is important. Consider the problems on the second exam:

- (1) This was a collection of definitions and examples used in lecture, homework and quizzes.
- (2) This was a slight variant of an example done in lecture (which itself was presented as an example from a Spring 2009 midterm).
- (3) (a) This was proven in class and was a writing problem. (Actually, this version was a bit easier because it only had two sets; the statement is true for any collection of open sets.)
(b) This is 11.7, a skills problem, with the base case provided for you.
- (4) (a) This specific type of problem was new, but the cardinality of these sets was discussed at length in class and homework. $\mathbb{R} \sim (0, 1)$ was the major conclusion of Takehome Problem #1.
(b) The inclusion : $\mathbb{N} \rightarrow \mathbb{Z}$, $h(n) = n$ works for this problem; inclusions were a major component of Takehome Problem #1.
(c) This was an example in lecture.

Note that I'm not implying you should memorize every problem done on homework or every example in lecture; the point is that if you've done all the homework, etc., and are comfortable with the ideas needed to solve those problems, you'll be able to handle anything on the exam. So don't spend hours and hours memorizing definitions and theorems; do problems instead, and you'll often find that you've learned the definitions and theorems just through using them.

Another technique for studying definitions and theorems is to come up with your own examples to learn why certain distinctions and conditions are important. For example, Theorem 17.1 requires $s_n \rightarrow s$ and $t_n \rightarrow t$. Can you show why that condition is important for part (a) of the theorem by showing how the conclusion might not be true if (s_n) and (t_n) diverge? In the Monotone Convergence Theorem, can you explain why it's important to know that the sequence is bounded?

SUGGESTED PROBLEMS

I highly recommend you redo any homework problems that you struggled with. Remember that most skills problems are only graded for completion, so the absence of written comments from your TA isn't a guarantee that they're correct; read through the solutions online to check them! Pay particular attention to those problems I mentioned in class as "typical exam problems," like Exercise 18.3. Anything important enough to have a mathematician's name attached to it – like Cauchy – is worth knowing.

In each section, the true/false questions at the beginning of the section are good way to refresh your memory about the section. In addition I'd suggest the following as review problems to start with (there's some overlap with previously assigned problems). At first you should use your textbook, notes and other resources if you're stuck, but your eventual goal is to be able to solve these problems without using any help.

§16: 16.6(a,c), 16.8, 16.9

§17: 17.3, 17.5(c,e,g,i,k), 17.6, 17.15(a), 17.16

§18: 18.3(b,c,e), 18.5, 18.6, 18.7

§32: 32.3(a), 32.4(a,c), 32.5(d,i,k)