

The following is a non-comprehensive list of solutions to the skills problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

- 2.4 (a) No basketball players at Central are short.
 (b) At least one light is off.
 (c) Every bounded interval contains infinitely many integers.
 (d) $\forall x \in S, x < 5$.
 (e) $\exists x \in (0, 1) \ni 2 \leq f(x) \leq 5$
 (f) $x > 5$ and $\forall y > 0, x^2 \leq 25 + y$
- 3.3 (a) If there exists a non-blue violet, then there exists a non-red rose.
 (b) If H is regular then H is not normal.
 (c) If K is not compact, then K is not closed or K is not bounded.
- 3.6 (j) $x = 1$ serves as a counterexample; its reciprocal is $y = 1$, which is not less than 1.
 (k) $n = 5$ serves as a counterexample; $3^5 + 2 = 245$, which is divisible by 5.
 (l) $x = 0$ serves as a counterexample; $0 \in \mathbb{Q}$ and $x = 0$ satisfies the equation.
 (m) $x = -1$ serves as a counterexample; $-1 \in \mathbb{Q}$ and $x = -1$ satisfies the equation.
- 3.7 (f) I'd suggest proving the (logically equivalent) contrapositive statement, "If p is even or q is even, then pq is even." First suppose p is even, so $p = 2n$ for some integer n . Then $pq = 2nq = 2(nq)$, which has the form of an even integer. Hence pq is even, as desired. (This doesn't yet complete the proof yet. We've just done the case where p is even, but it might be q which is the even integer, not p . The proof of that case works almost exactly the same way.)
 (h) As with (f), I'd suggest proving the contrapositive, "If p is even, then p^2 is even." Let p be an even integer, so $p = 2n$ for some integer n . Then $p^2 = 4n^2 = 2(2n^2)$, which is even.
- 4.12 (a) This proof is largely ok, although it might be nice to specifically mention the second "implied" case – what if $x = 0$?
 (b) This is a proof of the converse; it only shows that if $x = 0$ or $y = 0$, then $xy = 0$. It doesn't rule out the possibility that there are other numbers whose product could be zero.
- 4.13 (a) Let $x = a/b$ and $y = c/d$ be rational numbers, so $a, b, c, d \in \mathbb{Z}$ and $b, d \neq 0$. Then $x + y = \frac{ad+bc}{bd}$ is also a rational number; its numerator and denominator are integers and $bd \neq 0$.
 (b) Let $x = a/b$ and $y = c/d$ be rational numbers, so $a, b, c, d \in \mathbb{Z}$ and $b, d \neq 0$. Then $xy = \frac{ac}{bd}$ is also a rational number; its numerator and denominator are integers and $bd \neq 0$.