§1 Logical Connectives

Math consists of statements, sentences which can be classified as true/false - although we might not know which!

Ex Which are statements?

p: \(2+2=4\) yes - true!
q: \(3+3=10\) yes - false!

r: this statement is false \textbf{No}.

s: It's cold outside yes - assuming \textbf{cold} is defined

t: Truth is beauty. \textbf{No}.

u: \(x^2-4x+3=0\). Yes - truth value depends on value of \(x\)
Given statements $p, q$ we can create new ones using logic operators:

1. **Negation ($\neg$, $\sim$)**
   - $\neg p$ is true when $p$ is false,
   - false when $p$ is true.
   - Can represent as a "Truth Table":
     
     \[
     \begin{array}{c|c}
     p & \neg p \\
     \hline
     T & F \\
     F & T \\
     \end{array}
     \]

2. **Conjunction ($\land$, and)**
   - $p \land q$ is true when both $p$ and $q$ are true,
   - otherwise it's false.
   - Can represent as a "Truth Table":
     
     \[
     \begin{array}{c|c|c}
     p & q & p \land q \\
     \hline
     T & T & T \\
     T & F & F \\
     F & T & F \\
     F & F & F \\
     \end{array}
     \]
3. Disjunction (\( \lor \), or)

- \( p \lor q \) true if
  - \( p \) is true, \( q \) is true or both

⚠️ Rarer: Exclusive or (\( \lor, \text{xor} \))

- \( p \text{xor} q \) true if \( p \) or \( q \) is T but not both.

Ex
- \( p \): Jim is tall
- \( q \): Jim has red hair

\[ p \land q \]: Jim is tall and has red hair.
\[ \neg(p \land q) \]: NOT (Jim is tall and has red hair)

Don't write
- \( p \lor q \): Jim is not tall or Jim doesn't have red hair.
- \( \neg(p \lor q) \): Jim is not tall or Jim doesn't have red hair.
You try: truth table for \((\neg p) \lor (\neg q)\)

⚠️ \(\neg (p \land q)\) is T/F precisely when \((\neg p) \lor (\neg q)\) is T/F.
We say these statements are logically equivalent. This is one of De Morgan’s Laws:

\[
\neg (p \land q) = (\neg p) \lor (\neg q)
\]

In Words:
4. Implications (⇒, if..., then...) 
If p, then q. \( \boxed{p \Rightarrow q} \) 

p: antecedent (hyp.)
q: consequent (conclusion)

Mathematicians use following convention: \( p \Rightarrow q \) false only if p true and q is false. Otherwise it's true.

Ex. Determine truth values:

- If 2 is positive, then 4 is even. 
  - T
- If 3 is odd, then pigs can fly. 
  - F
- If pigs can fly, then I'm a rockstar. 
  - T
If \( p \implies q \) is true, and \( q \implies p \) is true, we write

\[ p \iff q \]

This is shorthand for "\( p \) and \( q \) are logically equivalent:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \implies q )</th>
<th>( q \implies p )</th>
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We also write: