

The third exam picks up where the second left off, covering Sections 4.1, 4.2, 4.3, 8.1 and 8.2. Any problems related to Section 8.2 will be limited to material covered in lecture on Friday, 12/2, which amounts to the Test for Divergence (which is really an 8.1 concept) and the Comparison Test. Overall that doesn't sound like much compared to the 6-8 sections covered on each of the previous midterms, but these sections are very dense and have a lot of material. Don't get caught thinking "I learned about sequences and series in my Calculus course, so I don't need to study as much for this test."

To study for this test you should learn the definitions and theorems, but **there is no substitute for doing problems**. You might be able to look at a sequence and determine its limit using methods from Calculus, but if you haven't worked out limits using the precise methods of Section 4.2, you will be hard pressed to do so on the test. You might be able to write down the definition of  $s_n \rightarrow s$  from Section 4.1, but if it's been three weeks since you used that definition to prove a certain sequence converges, it will be hard on the exam. If you had a high level calculus course, you may have learned the Monotone Convergence Theorem in a previous class – but in this class you need to be able to both use it and prove that it's true.

If you're nervous about proving things or using  $\varepsilon - N$  definitions on the exam, let me state again that we are not looking to surprise you with any of the problems on the test. Lectures, homework assignments and writing quizzes give you a good idea of what I feel is important. Consider the problems on the second exam:

- (1) This was a collection of definitions and examples used in lecture, homework and quizzes, including some that I announced beforehand would be on the exam.
- (2) This was a writing exercise and a writing quiz problem.
- (3) This was also a writing exercise and quiz problem.
- (4) This was a variant of a writing exercise (§2.4 #4)
- (5) This was a variant of something proved in class, and was on the review sheet.
- (6) This was a slight variant of an example done in lecture (which itself was presented as an example from a Spring 2009 midterm).

Note that I'm not implying you should memorize every problem done on homework or every example in lecture; the point is that if you've done all the homework, etc., and are comfortable with the ideas needed to solve those problems, you'll be able to handle anything on the exam. So don't spend hours and hours memorizing definitions and theorems; do problems instead, and you'll often find that you've learned the definitions and theorems just through using them.

Another technique for studying definitions and theorems is to come up with your own examples to learn why certain distinctions and conditions are important. For example, Theorem 4.2.1. requires  $s_n \rightarrow s$  and  $t_n \rightarrow t$ . Can you show why that condition is important for part (a) of the theorem by showing how the conclusion might not be true if  $(s_n)$  and  $(t_n)$  diverge? In the Monotone Convergence Theorem, can you explain why it's important to know that the sequence is bounded?

Keep in mind that if a statement about sequences isn't true, you can usually find a pretty simple counterexample. If you look at §4.2 #6, for example, I'd suggest that a "toolbox" of simple sequences like  $a_n = 1$ ,  $b_n = (-1)^n$ ,  $c_n = n$ , and so on, are usually enough.

## SUGGESTED PROBLEMS

I highly recommend you redo any homework problems that you struggled with. This applies to skills problems as well – even if we stress the writing problems in this class, the skills problems are there because they cover skills necessary for the writing problems. Pay particular attention to those problems I mentioned in class as “typical exam problems,” like Exercises 3 or 7 in §4.3. Anything important enough to have a mathematician’s name attached to it – like Cauchy – is worth knowing.

In each section, the true/false questions at the beginning of the section are good way to refresh your memory about the section. In addition I’d suggest the following as review problems to start with. At first you should use your textbook, notes and other resources if you’re stuck, but your eventual goal is to be able to solve these problems without using any help.

**IN PARTICULAR**, although we post lots of solutions to help you evaluate your work, you should **NOT** just read through those solutions, verify they make sense, and think that you’re prepared for the exam. This is a common mistake; there is a huge difference between reading and understanding our work, and being able to produce your own proofs and solutions on an exam with no books, notes, or solutions in front of it. To repeat: your goal is to be able to solve these problems without any outside resources. This requires practice!

In addition to previously assigned problems, here are some suggested problems to work on; I haven’t gone through with a fine toothed comb to remove every problem on previous homework sets, so there will be some overlap.

§4.1: 6(b,d,f), 7(b,c), 8(a,b)

§4.2: 3(b), 5(e,g,i), 7, 8, 19(a,c)

§4.3: 3(b,e), 4, 5, 6, 7

§8.1: 3(a), 4(a,b), 5(d,e,j), 8

§8.2: 3(c,f,l)

If you’d like more studying advice, it’s worth going back and re-reading the Exam 2 debrief, if you haven’t done so yet:

<http://www.math.umn.edu/~rogness/math3283w/f16docs/3283-f16-exam2-debrief.pdf>