From our previous "abstract" example...

\[ R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (c,b), (b,a), (b,c), (a,c), (c,a)\} \]

\[ aRa \quad dRd \quad aRb \quad \ldots \]

Equivalence classes:

\[ E_a = \{a, b, c\} \]

\[ E_d = \{d\} \]

So \( R \) "partitions" set \( S = \{a, b, c, d\} \) into two disjoint subsets.
Proposition Different Equiv. classes are disjoint.

If \( E_x \) and \( E_y \) are equiv. classes for eg rel'n \( \sim \),
then \( E_x \cap E_y = \emptyset \) or \( E_x = E_y \).

Pf: Let \( x, y \in S \) with eg rel'n \( \sim \).

If \( E_x \cap E_y = \emptyset \), we're done!

Else \( z \in E_x \cap E_y \). We can show \( E_x \subseteq E_y \) and
\( E_y \subseteq E_x \) which means \( E_x = E_y \).

Suppose \( z \in E_x \). Then \( z \sim x \sim w \sim y \) (blc \( w \in E_x \) and \( w \in E_y \)).
By transitivity, \( z \sim y \Rightarrow z \in E_y \Rightarrow E_x \subseteq E_y \).

(\( E_y \subseteq E_x \) similar)
Def: A partition of a set $S$ is a collection $P$ of non-empty subsets of $S$ such that

(a) $\forall x \in S, \exists A \in P \text{ s.t. } x \in A$
(b) $\forall A, B \in P$, either $A \cap B = \emptyset$ or $A = B$.

Ex: $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3\} \quad B = \{4, 6, 8, 10\} \quad C = \{5\} \quad D = \{7, 9\}$

$\Rightarrow P = \{A, B, C, D\}$ partition $S$.

Ex: Assigning 6th graders to soccer teams

(a) says everybody is on a team
(b) says no student on two different teams.
Key idea: Equivalence Classes Form a Partition

Any elt $x$ in a set $S$ with $\sim$ belongs to an eq class ($[x]$), and the classes are disjoint ($\text{Prop'}n$)

$\text{Ex } S = \text{UMN Students, } xRy \iff x, y \text{ born in same year}$

$R$ is an equiv. reln (you check)

$E_{1993} = \{\ldots \ldots 1993\}$

$E_{1994} = \{\text{people born in 1994}\}$

$E_{1995} = \{\ldots \ldots 1995\}$

$\vdots$

$\mathcal{P} = \{E_{1900}, \ldots, E_{2016}\}$ form a part of student body.
Thm 2.2.17

(a) If $S$ has eq reln $R$, the eq classes form a partn of $S$.

(b) If $P$ is a partn of $S$, the relation $xRy \iff x, y$ are in same set of partn is an equivalence relation.

Ex $S$

\[ \begin{array}{cccccc}
  a & b & c & d \\
  i & h & g & e \\
\end{array} \]