The following is a *non-comprehensive* list of solutions to the computational problems on the homework. For some problems there is a sketch of a solution. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work. Feel free to talk to us about any of the writing intensive problems.

Please let us know if you spot any typos and we'll update things as soon as possible.

2.3.26: There are lots of possible answers. Here is one:

$$f : \mathbb{N} \to \mathbb{R}, \quad f(n) = n$$
$$g : \mathbb{R} \to \mathbb{R}, \quad g(x) = x^2$$

The function $g$ is not injective because $g(-1) = g(1)$, but $f$ is injective, and the composition

$$g \circ f : \mathbb{N} \to \mathbb{R}, \quad g \circ f(n) = n^2$$

is also injective. (Because $-1$ is not in the domain, we can't have $g \circ f(-1) = g \circ f(1)$, for example.)

2.3.27: There are lots of possible answers. Here is one, where $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$.:

$$f : \mathbb{N}_0 \to \mathbb{Z}, \quad f(n) = n$$
$$g : \mathbb{Z} \to \mathbb{N}_0, \quad g(m) = |m|$$

In this case $f$ is not surjective, because its output does not include the negative integers. But $g$ is surjective, and $g \circ f(n) = g(f(n)) = |n| = n$ is the identify function from $\mathbb{N}_0$ to $\mathbb{N}_0$, which is surjective.

2.3.28: The example I used in problem 27 works here, too!

2.4.3: Talk to us if you have trouble trying to example why any of these functions are bijections. There are many possible answers for each question!

(a) $f : [0, 1] \to [1, 4]$, $f(x) = 3x + 1$.

(b) $f : [0, 1] \to [2, 7]$, $f(x) = 5x + 2$.

(e) $f : (0, 1) \to (0, \infty)$, $f(x) = -\ln(x)$.

(f) $f : (0, 1) \to \mathbb{R}$, $f(x) = \tan(\pi x - \pi/2)$.