The following is a *non-comprehensive* list of solutions to the computational problems on the homework. For some problems there is a sketch of a solution. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work. Feel free to talk to us about any of the writing intensive problems.

Please let us know if you spot any typos and we'll update things as soon as possible.

(1) Do the True / False problems at the beginning of the 2.4 exercises.

Answers to 2.4.1: T (definition); T (definition); F (the actual term is *transfinite*, but that's not a crucial term for us); F (the bijection should be $\mathbb{N} \to S$); T (Theorem 2.4.9); F (it might be finite, e.g. $\{1, 2\} \subseteq \mathbb{N}$.

Answers to 2.4.2: F (Look at Theorem 2.4.10. It should be an injection $S \to \mathbb{N}$ or a surjection $\mathbb{N} \to S$); T (proved in class); T (Theorem 2.4.10); F (we proved it's uncountable); T (by definition); T

(2) Find a specific bijection between $(0, 1)$ and $[0, 1)$ to show the sets are equinumerous. Explain why your function is a bijection.

One possibility is to modify the bijection we constructed in class from $(0, 1)$ to $(0, 1]$. Here's a function $f : (0, 1) \to [0, 1)$ which could work:

$$f(x) = \begin{cases} 1 - \dfrac{1}{2^{n-1}}, & x = 1 - 2^n, \text{ some } n \in \mathbb{N} \\ x, & \text{otherwise} \end{cases}$$

Thus $f(1/2) = 0$, $f(3/4) = 1/2$, $f(7/8) = 3/4$, $f(15/16) = 7/8$, and so on. Talk to us if you're not sure why this is a bijections.

(3) Let $\mathscr{P}(S)$ be the power set of S. Determine whether each of the following is True of False. Explain your answers.

(a) For every set S, $\emptyset \subseteq \mathscr{P}(S)$.

True. The empty set is a subset of every set.

(b) For every set S, $\emptyset \in \mathscr{P}(S)$.

Also true. The elements of $\mathscr{P}(S)$ are the subsets of S, and $\emptyset$ is guaranteed to be one of them. For example, $\mathscr{P}(\{2,3\}) = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$.

(c) $\{2\} \subseteq \mathscr{P}(\{2,3\})$

False. $\{2\}$ is an element of $\mathscr{P}(S)$, but not a subset.

(d) $\{2\} \in \mathscr{P}(\{2,3\})$

True. Look at $\mathscr{P}(\{2,3\})$ written out above. Notice that $\{2\}$ is one of the elements in that power set.

(e) $\{\{2\}\} \subseteq \mathscr{P}(\{2,3\})$. Every element of the set on the left (there's only one, namely $\{2\}$) is an element of the set on the right.

True!

This problem can be tricky because of the notation. It might help to rename some of the elements in $\mathscr{P}(\{2,3\})$. Instead of $\{\emptyset, \{2\}, \{3\}, \{2,3\}\}$, let's write the elements as $\{0, a, b, c\}$. Then parts (c)–(e) amount to:

$$a \subseteq \mathscr{P}(\{2,3\}) \text{ (false)}$$
$$a \in \mathscr{P}(\{2,3\}) \text{ (true)}$$
$$\{a\} \subseteq \mathscr{P}(\{2,3\}) \text{ (true)}$$