Math 3283W Fall 2016 - Writing Quiz #4
Thursday 10/13/16 - 20 minutes **Name:**

Remember: your work on in the "writing" portion of this quiz will be graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

Definitions. This portion of your quiz will be graded for mathematical correctness only.

(1) (3 Points) Complete this definition: a function $f : A \to B$ is *injective* if...

whenever $f(a) = f(a')$, we have $a = a'$.

OR $a \neq a' \Rightarrow f(a) \neq f(a')$

(2) (3 Points) Complete this definition: a function $f : A \to B$ is *surjective* if...

For every $b \in B$, there exists $a \in A$ such that $f(a) = b$.

(or equivalent)

Writing. Recall that a function is a *bijection* if it is both injective and surjective.

(3) (9 Points) Suppose $f : A \to B$ and $g : B \to C$ are both bijections. Prove $g \circ f$ is also a bijection. (You must prove $g \circ f$ is an injection and surjection, and not simply cite results about compositions of injective or surjective functions.)

We must show $g \circ f$ is injective and surjective.

injective Suppose $a \neq a'$ in A. Then $f(a) \neq f(a')$ because f is injective. Similarly, g is injective, so $g(f(a)) \neq g(f(a'))$, i.e. $g \circ f(a) \neq g \circ f(a')$. Hence $g \circ f$ is injective.

surjective Let $c \in C$. Because g is surjective, there exists $b \in B$ such that $g(b) = c$. The function f is also surjective, which means there is an $a \in A$ for which $f(a) = b$. Now

$$g \circ f(a) = g(f(a)) = g(b) = c.$$

Thus $g \circ f$ is surjective.