Definitions. This portion of your quiz will be graded for mathematical correctness only.

(1) (3 Points) Complete this definition: a function \( f : A \to B \) is injective if...

\[
\text{whenever } f(a)=f(a'), \text{ we have } a=a'.
\]

(2) (3 Points) Complete this definition: a function \( f : A \to B \) is surjective if...

\[
\text{For every } b \in B, \text{ there exists } a \in A \text{ such that } f(a)=b.
\]

(or equivalent)

Writing. Recall that a function is a bijection if it is both injective and surjective.

(3) (9 Points) Suppose \( f : A \to B \) and \( g : B \to C \) are both bijections. Prove \( g \circ f \) is also a bijection. (You must prove \( g \circ f \) is an injection and surjection, and not simply cite results about compositions of injective or surjective functions.)

We must show \( g \circ f \) is injective and surjective.

\[\text{Injective} \quad \text{Suppose } a \neq a' \text{ in } A. \text{ Then } f(a) \neq f(a') \text{ because } f \text{ is injective. Similarly, } g \text{ is injective, so } g(f(a)) \neq g(f(a')). \text{ i.e. } g \circ f(a) \neq g \circ f(a'). \text{ Hence } g \circ f \text{ is injective.} \]

\[\text{Surjective} \quad \text{Let } c \in C. \text{ Because } g \text{ is surjective, there exists } b \in B \text{ such that } g(b)=c. \text{ The function } f \text{ is also surjective, which means there is an } a \in A \text{ for which } f(a)=b. \text{ Now } \]

\[g \circ f(a) = g(f(a)) = g(b) = c.\]

Thus \( g \circ f \) is surjective.