Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. You should write in complete sentences with reasonably correct grammar. Math 5335 is not a writing intensive course, but it *is* a 5000-level mathematics course, and you're expected to be able to explain your work in a coherent, organized and logical manner.

## Homework Assignment

**Section 2.5:** Problems 3, 12, 13 and 16.

**Extra Problem:** At the end of Chapter 2, based on our definition of the measure of an angle, we defined the arccos function. The goal of this problem is to construct its inverse, the cosine function, and the develop the "geometric definition" of the dot product which I have often referred to in class. For part (a) you may wish to review inverse functions in a calculus textbook, or talk to me during office hours.

a Draw a picture of the function $f(t) = 1/\sqrt{1-t^2}$, $t \in [-1, 1]$. Use this sketch and the definition of $\arccos(z)$ to explain why

$$\arccos : [-1, 1] \to [0, \pi]$$

has an inverse function, which we will call cos:

$$\cos : [0, \pi] \to [-1, 1]$$

b Prove the following fact about the dot/scalar/inner product, which we have often used in class:

$$\langle aU, bV \rangle = ab\langle U, V \rangle$$

c Let $U$, $V$ be unit vectors in the same direction as $A$ and $B$. Prove $\langle U, V \rangle = \frac{\langle A,B \rangle}{||A|| \cdot ||B||}$.

d Let $p$, $r$ be rays from a common vertex with direction indicators $A$ and $B$, respectively. Let $\theta$ denote the measure of $\angle(p, r)$. Use part (c) to show

$$\theta = \arccos\left(\frac{\langle A, B \rangle}{||A|| \cdot ||B||}\right)$$

Conclude that $\cos\theta = \frac{\langle A,B \rangle}{||A|| \cdot ||B||}$ or, equivalently, $\langle A, B \rangle = ||A|| \cdot ||B|| \cos\theta$.