The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

8.4.3: You can almost use the formula given in the book for a circle inversion, with one exception: that formula assumes the mirror is centered at the origin, whereas our circle is centered at $(-8, 13)$. So we need to first move everything so the center is at the origin, then invert, and then move it back:

$$\mathcal{I}(X) = \begin{cases} \frac{\rho^2}{||X-C||^2}(X-C)+C, & X \neq C, \infty \\ \infty & X = C \\ C & X = \infty \end{cases}$$

where $C = (-8, 13)$ and $\rho = 29$ in our case. By my quick calculations, this yields:

$$\mathcal{I}(0,0) = \left(\frac{4864}{233}, \frac{-7904}{233}\right)$$

$\mathcal{I}(12,-8) = (12,-8)$ (this point is on the mirror, so it stays fixed!)

$$\mathcal{I}(\infty) = (-8, 13)$$

$$\mathcal{I}(8,-13) = \left(\frac{1500}{233}, \frac{-4875}{233}\right)$$

8.4.4: As we apply Theorem 4, $\rho = 1$ because it's the radius of the mirror.

(i) Using Case (iv) of the theorem, with $C = (0, 0)$ and $r = 2$, we get

$$||\mathcal{I}(X) - (0,0)|| = \frac{2}{|-4|}$$
$$||\mathcal{I}(X)|| = \frac{1}{2}$$

which is a circle of radius 1/2 centered at the origin.

(ii) Here we get a circle of radius 1/3 centered at the origin, similar to the first part.

(iii) Case (iii) of the theorem, with $C = (0, -1)$ and $r = 1$. This results in

$$\langle \mathcal{I}(X), (0,-1)\rangle = \frac{1}{2}$$

In other words, we have the line $\langle (x, y), (0, -1)\rangle = \frac{1}{2}$ or $y = -1/2$.

(iv) This is the line $y = -1$, which is not through the origin. Since $(0, 1)$ is perpendicular to the line, I could write it as $\langle (x, y), (0, 1)\rangle = -1$, and we're in case (ii) of the theorem with $A = (0, 1)$ and $b = -1$. The resulting circle has the equation

$$||\mathcal{I}(X) + (1/2)(0,1)|| = \frac{1}{2}(1)$$
$$||\mathcal{I}(X) - (0,-1/2)|| = \frac{1}{2}$$

That's a circle of radius 1/2 centered at $(0, -1/2)$

(v) The line $y = x$ goes through the origin, so its image under the reflection across the mirror is the same line, by part (i) of the theorem.

(vi) This is Case (iv) of the theorem with $C = (-3, 0)$ and $r = 5/2$. The result is

$$||\mathcal{I}(X) - \frac{1}{9 - 25/4}(-3, 0)|| = \frac{5/2}{|9 - 25/4|}$$
$$||\mathcal{I}(X) - (-33/4, 0)|| = \frac{10}{11}$$

A circle of radius 10/11 centered at $(-33/4, 0)$.

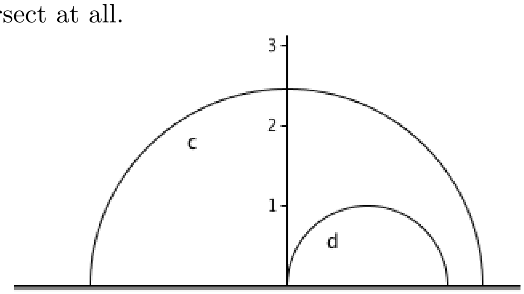
8.4.9: By measuring corresponding side lengths, we see that the larger triangle has side lengths which are 3 times longer than the smaller one. (So its area is 9 times larger.) After scaling it down by 3 units ($\rho = 1/3$ in the formula for the conformal affinity) you have a triangle which is congruent to the other one. You can find the isometry which sends one to the other using the procedure in Chapter 7 (where you combine reflections to map one congruent triangle to another). Ask me if you had trouble figuring this out.

9.9.3: This problem is trickier than it appears. Here's a plausible proof: let P be a point, and choose Q and R so that P, Q and R are not collinear. Then P is not on the line containing Q and R.

Can you spot that fallacy in that proof? Axiom I.3 guarantees that there exist three noncollinear points. It doesn't guarantee that some random point P is one of them! So more care is required.

Let P be a point, and let A, B and C be three noncollinear points. I claim that P can't possibly be incident with all three of $j = \overleftrightarrow{AB}$, $k = \overleftrightarrow{BC}$ and $l = \overleftrightarrow{AC}$. Suppose it *were* incident with all three. Then I.1 says j is the unique line which contains P and B – but so is k! So $j = k$, and similarly you could show $k = l$. Since the whole point of j, k and l is that they are distinct lines, this is nonsense, and P can't be incident with all of them. Equivalently, there is at least one of them which is not incident with P.

9.9.7: These lines don't intersect at all.



9.9.22: The (Poincare) line through $(-3, 5)$ and $(-3, 2)$ is the vertical ray $x = -3$, $y > 0$. The (Poincare) line through $(2, 5)$ and $(5, 4)$ is a (Euclidean) semicircle – specifically, using equations 9.6 and 9.7, the semicircle with center $(2, 0)$ and radius 5:

$$(x - 2)^2 + y^2 = 25$$

9.9.23: Let $A = (2, 4)$, $B = (-2, 4)$ and $C = (0, 4)$. By symmetry, the line $\overleftrightarrow{AB}$ must be centered at the origin and have radius $\sqrt{2^2 + 4^2} = \sqrt{20}$:

$$x^2 + y^2 = 20$$

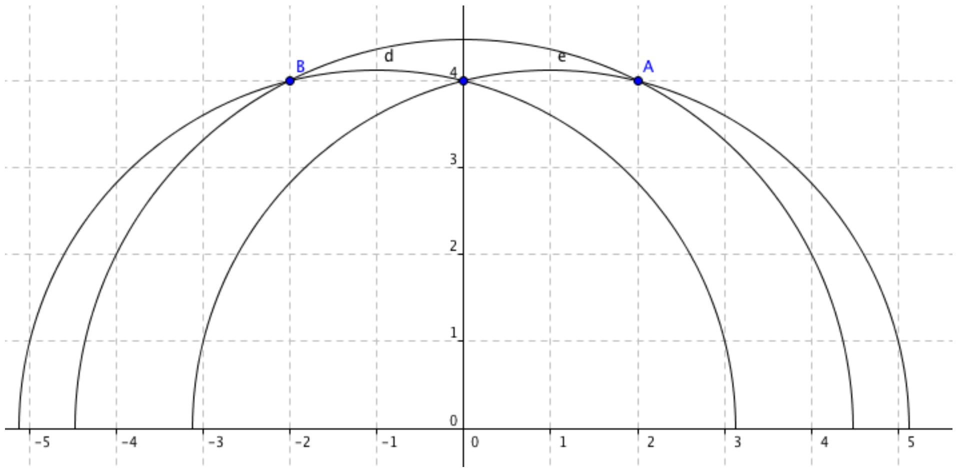
Using the same formulas as in #22, we find $\omega = 1$ and $\rho = \sqrt{(17)}$ for $\overleftrightarrow{AC}$:

$$(x-1)^2 + y^2 = 17$$

By symmetry, the line $\overleftrightarrow{BC}$ must be

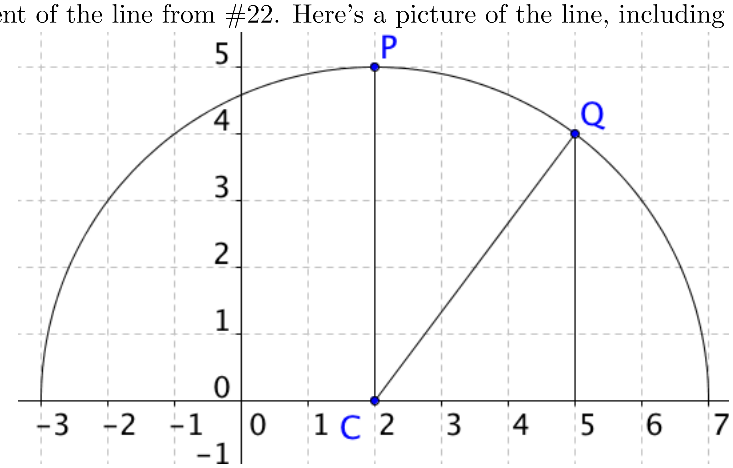
$$(x+1)^2 + y^2 = 17$$

These lines are shown below; the triangle is the top region.



9.9.25: (i) $\ln(5/2) = |\ln(2/5)|$

(ii) This is a segment of the line from #22. Here's a picture of the line, including the points.



If $t_1 = |\angle PC(5,0)|$ and $t_2 = |\angle(5,0)CQ|$, then you can read off the values of trig functions as needed from the picture, e.g. $\sin t_2 = 4/5$ so $\csc t_2 = 4/5$, etc. Working through the distance formula gives $\ln 2$.

9.9.26: The work here is very similar to 9.9.25 and the answers are given in the back of the book.