The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an
answer with just a few words of explanation. On other problems the stated solution may be complete. As
always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I’ll update things as soon as possible.

Chapter 10: Answers to 10.24, 10.25, and 10.26 are in the back of the book; I’ve included them here:

10-24. \( x^2 + y^2 = 20, (x - 1)^2 + y^2 = 17, (x + 1)^2 + y^2 = 17 \) (see Figure A.6).

Figure A.6: Problem 9-24: the Poincaré triangle with given vertices \( Q = (2, 4), P = (-2,4), R = (0, 4). \)

10-25. (i) \( \ln(5/2); \) (ii) \( \ln 2 \)

10-26. one side of length \( \ln[(3 + \sqrt{5})/2] \) and two sides of length \( \ln[(9 + \sqrt{17})/8] \). These latter two
sides are shorter than the first side since \( 3/8 < 3/2 \) and \( \sqrt{17} < 4\sqrt{5} \). The sum of the lengths of the two
shorter sides is

\[
\ln\left[\left(\frac{9 + \sqrt{17}}{8}\right)^2\right] = \ln\left(\frac{49 + 9\sqrt{17}}{32}\right).
\]

That the triangle inequality is satisfied can be seen from the fact that \( \frac{49}{32} > \frac{3}{2} \) and \( 9\sqrt{17} > 16\sqrt{5} \),
this latter fact following from \( 81 \cdot 17 > 256 \cdot 5 \).

11.3: We’re given direction indicators \((-5, 0), (2, 0)\) and \((\infty, 0)\). A line is directed by exactly two
direction indicators, so we can form three different lines here:

- \( x = -5 \) is directed by \((-5, 0)\) and \((\infty, 0)\)
- \( x = 2 \) is directed by \((2, 0)\) and \((\infty, 0)\)
- \( \left(x + \frac{3}{2}\right)^2 + y^2 = \left(\frac{7}{2}\right)^2 \) is directed by \((-5, 0)\) and \((2, 0)\).

11.5: In the following picture, \( c \) is the line \( x^2 + y^2 = 4 \), which is directed by \((-2, 0)\) and \((2, 0)\). \( j: x = 2 \)
is asymptotically parallel to \( c \) because it’s directed by \((2, 0)\) and \((\infty, 0)\), so it shares exactly one di-
rection indicator with \( c \) (and includes \( P = (2, 2) \)). Similarly, \( k: (x - 1/2)^2 + y^2 = (5/2)^2 \) contains
\( P = (2, 2) \) and is directed by \((-2, 0)\) (shared with \( c \)) and \((3, 0)\). You can find this by luck (i.e.
guessing and checking) or the formula in Proposition 11.4.

The line \( m : x = 3 \) is directed by \((3, 0)\), which it shares with \( k \), and \((\infty, 0)\), which it shares with \( j \).
11.7: I’ve included pictures of the lines below; let me know if you have difficulty finding the equations of any of them:

- \( x^2 + y^2 = 9 \)
- \( x^2 + y^2 = 1 \)
- \((x + 1)^2 + y^2 = 4\)
- \((x - 1)^2 + y^2 = 4\)

11.8: We can use Theorem 11.7 with \((g, h) = (0, 5), (a, 0) = (5, 0)\) and \((u, 0) = (-10 - 5\sqrt{3})\). A bunch of algebra (*cough*, *cough*, Wolfram Alpha) shows

\[
\frac{h^2 + (a - g)(u - g)]^2 - h^2(u - a)^2}{[(a - g)^2 + h^2][(u - g)^2 + h^2]} = \cdots = -\frac{1}{2}
\]

with those values of \(g, h, a\) and \(u\). Thus

\[
\theta = \arccos \left(-\frac{1}{2}\right) = \frac{2\pi}{3}
\]

A: If you’re not sure how to do this problem, it would be best to stop by my office sometime. We’ll draw on a sphere and talk through it. But here’s the basic idea. The picture below shows a spherical triangle with angles \(\alpha, \beta\) and \(\gamma\). The shaded region of the sphere is the double lune with angle \(\alpha\). Because the entire sphere (of radius 1) has surface area \(4\pi\), this double lune has area

\[
2 \cdot \frac{\alpha}{2\pi} \cdot 4\pi = 4\alpha.
\]
The “magic” happens when you shade in the double lunes for $\alpha$, $\beta$ and $\gamma$. (Use different colors!) You shade the entire sphere in; most of the sphere is covered with one color, but the triangle will be shaded by all three colors. Meanwhile, there’s a congruent copy of the triangle on the opposite side of the sphere. (See it in the picture above? It’s in the back, and upside down, compared to the original triangle.) That extra copy of the triangle is the only other place which is shaded by more than one color. Like the original triangle, it’s actually shaded with all three of them.

What that means is that, if you add the area of the three double lunes together, you get the area of the entire sphere, plus some overcounting. Each of the triangles (front and back) has been shaded three times, which means it’s been overcounted twice. All in all, we’ve overcounted the area of the triangle four times:

$$4\alpha + 4\beta + 4\gamma - 4\|\triangle ABC\| = 4\pi$$

A bit of simplification yields the final formula:

$$\|\triangle ABC\| = (\alpha + \beta + \gamma) - \pi$$