Functions

Let $A$ and $B$ be sets. Recall that a function $f : A \to B$ is a rule which assigns a value $f(a) \in B$ for every $a \in A$. The function $f$ is:

**Surjective** (or *onto*) if it “hits” everything in $B$; that is, given any element $b \in B$, there is at least one $a \in A$ such that $f(a) = b$. (There might be more than one.)

**Injective** (or *one-to-one*) if every element $a \in A$ is sent to a different element in $B$. In other words, if $a_1$ and $a_2$ are different elements of $A$, then $f(a_1) \neq f(a_2)$.

Determine whether the following functions are surjective and/or injective. If you think something is not a valid function, explain why.

1. $f : \{1, 2, 3\} \to \{4, 5\}$ where $f(1) = 4$, $f(2) = 5$, and $f(3) = 4$.

2. $f : \mathbb{N} \to \mathbb{N}$ where $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ (i.e. the natural numbers) and $f(n) = n + 1$.

3. $f : \mathbb{Z} \to \mathbb{Z}$ where $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ (i.e. the integers) and $f(x) = 2x$.

4. $f : \mathbb{Q} \to \mathbb{Q}$ where $\mathbb{Q} = \{\frac{p}{q} \text{ where } p \text{ and } q \text{ are integers and } q \neq 0\}$ (i.e. the rational numbers) and $f(x) = 2x$. 
5. \( f : \mathbb{R} \to \mathbb{R}^2 \) where \( f(x) = (x, 1) \).

6. \( f : \mathbb{R} \to \mathbb{R}^2 \) where \( f(x) = (x, x^2) \).

7. \( f : \{ \text{University Students} \} \to \{ \text{7 digit numbers} \} \), \( f(S) = S \)'s student ID number.

8. \( f : \{1, 2, 3, \ldots, 21\} \to \{ \text{Pro Cyclists} \} \), \( f(n) = \text{Winner of Stage } n \) of the 2015 Tour de France.

9. \( f : \{ \text{Times of the day} \} \to \mathbb{R} \), \( f(t) = \text{temperature at time } t \).

10. \( f : \{ \text{Population of the world} \} \to \{ \text{Pet Names} \} \), \( f(P) = \text{name of } P \)'s dog.