Lines, Segments and Rays

1. Let $P$ and $Q$ be points. Draw a sketch of each of the following sets and describe them in words.

   (a) The line $\overrightarrow{PQ} = \{ P + s(Q - P) : s \in \mathbb{R} \}$.

   (b) The ray $\overrightarrow{PQ} = \{ P + s(Q - P) : s \geq 0 \}$.

   (c) The segment $\overline{PQ} = \{ P + s(Q - P) : 0 \leq s \leq 1 \}$.

   (d) Is $\overrightarrow{PQ} = \overrightarrow{QP}$? Is $\overrightarrow{PQ} = \overrightarrow{QP}$? Is $\overline{PQ} = \overline{QP}$?

2. For a vector $X$, we defined $\|X\| = \sqrt{\langle X, X \rangle}$. How would you define the length of a line segment $\overline{PQ}$?

3. The point $M = P + \frac{1}{2}(Q - P)$ is the midpoint of $\overline{PQ}$. Draw a sketch to illustrate this, and prove that the midpoint is “equidistant” from $P$ and $Q$, i.e. that the line segments $\overline{PM}$ and $\overline{QM}$ have the same length.