3.1 #3 (a) Show $P^2$ contains a Möbius band.

Use a std gluing diagram for $P^2$; the shaded subset $X$ will be homeomorphic to $M^3$.

(b) This was an example in class. The key is to label the edges of the disc in a "useful" manner.

Note the bodies of $M^3$ and the disc are both $b$ followed by $c$, so gluing these edges together gives the desired surface. To show the surface is $P^2$, actually put the $b$'s together.

(let $d = ca^{-1}$, $o \rightarrow o$)
31. #4  (a) Start at P and go clockwise:

   after gluing edges, the 4 quarter-circles become a disk:

   \[ \text{The interior of this disk is an open disc } (\mathbb{E}^2) \text{ containing the corner point of the rectangle.} \]

\[ (b) \quad \begin{array}{c}
\text{P = starting point of a, also the starting point of c, which also makes it the ending point of b, and that's it.}
\text{Q = starting pt of b, which also makes it the end of a and end of c.}
\end{array} \]

So the 6 vertices are glued into two groups of three.

(c) [Note: I showed you this gluing diagram in class or on an overhead, it gives a torus.]

\[ \begin{array}{c}
\text{cut along d, glue along b}
\end{array} \]

\[ e = a^{-1} c, \quad ve = T^2 \]
3.2 #1 (a) Think of the polygon as a subset of \( \mathbb{R}^2 \). By def'n if \( x \) is an interior pt, then \( B_r(x) \subset \text{Polygon for some } r > 0 \).

(b)

(c) This is like 3.1 #4(a), where you trace around vertices and get quarter circles which assemble into a disc. It may not always be quarters, but you will get a disc. I'm ok if you didn't do a rigorous proof that you eventually get back to where you started, etc.

3.2 #7 (a) Won't work; need to have both \( a, b \) on each triangle so you can glue them together along an edge other than \( c \), which is a Klein Bottle, and that's what's given as \( ab^{-1}b^{-1} \).

(b) By example 3.15, \( a \alpha_4 b = \alpha_3 \neq \alpha_2 \).
3.2 #8

(a) \( T^2 \# P^3 \) produces a similar result:

\[
abc^{-1}b^{-1} \quad \text{vs.} \quad aba^{-1}b^{-1} \quad \text{dd}
\]

(b) Start with \( T^2 \# P^3 \):

\[
\text{cut along } c \quad \text{glue along } d
\]

\[
\text{cut along } e \quad \text{glue along } b
\]

Relabeling gives:

\[
(x = e, \quad y = a^{-1}, \quad z = c)
\]

Which matches the pattern for studying the diagram of \( K^3 \# P^3 \) in (a).

(c) In (b), \( T^2 \# P^3 \equiv K^2 \# P^3 \) but we can't "cancel" the \( P^3 \)'s since \( T^2 \neq K^2 \).