1. (a) Ask us if you’re not sure what the pictures look like.
(b) \( \int_0^2 \int_0^3 \int_{-3}^{3-z/2} f(x, y, z) \, dx \, dy \, dz \)
(c) \( \int_0^2 \int_3^{-3-z/2} \int_3^{2} 2 - \frac{2}{3} y - z \, dy \, dz; \) other answers possible.

2. (a) Again, ask us about the picture. For (b),

\[
V(S) = \int_S 1 \, dV = \int_0^{2\pi/3} \int_0^{\pi/4} \int_0^{\rho^2 \sin \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = 2\pi \frac{64}{3} \int_{\pi/4}^{\pi/3} \sin \phi \, d\phi = \cdots = \frac{64\pi}{3} \left( \sqrt{2} - 1 \right)
\]

3. (i) This is the half cone \( z = \sqrt{x^2 + y^2} \) for \( 0 \leq z \leq 1 \).
(ii) \( \vec{n} = \vec{r}_u \times \vec{r}_v = \langle -u \cos v, -u \sin v, u \rangle \), and \( \vec{n}(1/2, \pi/2) = (0, -1/2, 1/2) \).
(iii) \( A(S) = \int_0^1 \int_0^{2\pi} |\vec{r}_u \times \vec{r}_v| \, dv \, du = \cdots = 2\pi \sqrt{2} \int_0^1 u \, du = \pi \sqrt{2} \).

4. (a) One possibility is a clockwise half circle.
(b) One possibility is a straight line segment, up the \( y \)-axis.
(c) The line integral over the segment on the \( y \)-axis is zero; the other part is positive; hence the entire thing is positive.
(d) One possibility is a counter-clockwise circle around the origin.

5. (a) Ask us about the picture.
(b) \( \int_C \vec{F} \cdot \, d\vec{r} = \int_{3\pi/4}^{7\pi/4} \langle -\sin^2 t, \cos^2 t \rangle \cdot \langle \sin t, -\cos t \rangle \, dt = \cdots = -\int_{3\pi/4}^{7\pi/4} \sin^3 t + \cos^3 t \, dt \). This eventually boils down to \( -5\sqrt{2}/3 \).

6. (a) Ask us about the picture
(b) Let \( x = u^3 \), \( y = v^3 \), and \( z = w^3 \). Then the area expansion factor is simply \( 27u^2 v^2 w^2 \). In \( uvw \)-space, the domain is the tetrahedron in the first octant, cut off by the plane \( u + v + w = 8 \).

The integral is

\[
V(S) = \int_S 1 \, dV = \int_0^8 \int_0^8 \int_0^{8-u-v} 27u^2 v^2 w^2 \, dw \, dv \, du
\]

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