1. Each of the following line integrals evaluates to zero. Explain why, using a geometric argument where possible.
   (a) $\int_C e^{\arctan(x^4)}\, y^3\, \cos(2y)\, ds$, where C is the straight line segment from $(10, 15)$ to $(10, -15)$.
   (b) $\oint_C \frac{x}{x^2+y^2+1}\, dx + \frac{y}{x^2+y^2+1}\, dy$, where C is the unit circle oriented counterclockwise.
   (c) $\int_C -e^{xy}\, y\, dx + e^{xy}\, x\, dy$, where C is segment of a line passing through the origin.
2. Consider the vector field $\mathbf{F}(x, y, z) = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$. If C is any path from $(0, 0, 0)$ to $\mathbf{a} = (a_1, a_2, a_3)$, show that $\int_C \mathbf{F} \cdot d\mathbf{r} = \mathbf{a} \cdot \mathbf{a}$.
3. Let $\mathbf{F} = \langle e^x \cos y,\ -e^x \sin y,\ 1\rangle$. Take out a sheet of paper. Draw two or three curves, each of which is made of a number of line segments, pieces of circles, etc. Make one of them closed. Compute exact values of the line integral of $\mathbf{F}$ along each of these curves. Make sure your answers are carefully justified.
4. Evaluate the line integral of the vector field $\mathbf{F}(x, y, z) = \frac{x\mathbf{i}+y\mathbf{j}+z\mathbf{k}}{(x^2+y^2+z^2)^{3/2}}$ over the path $\mathbf{x}(t) = \langle t, \sin t, \cos t\rangle$, $0 \le t \le 1$.
5. For each of the following sets, determine if any of these adjectives apply: open, closed, connected, disconnected, simply connected.
   (a) $A = \{(x, y) \mid x \ge 0, y \ge 0, x + y \le 1\}$
   (b) $B = \{(x, y) \mid x^2 + y^2 = 81\}$
   (c) $C = \{(x, y) \mid x < 0 \text{ or } x > 3\}$
   (d) $D = \{(x, y) \mid x^2 + y^2 < 1 \text{ or } (x - 3)^2 + (y + 4)^2 < 1\}$
   (e) $E = \{(x, y, z) \mid 4 < x^2 + z^2\}$
   (f) $F = \{(x, y, z) \mid |x| \ge 1 \text{ or } |y| \ge 1 \text{ or } |z| \ge 1\}$
   (g) $G = \{(x, y, z) \mid |x| \ge 1 \text{ and } |y| \ge 1 \text{ and } |z| \ge 1\}$
   (h) $H = \{(x, y, z) \mid |x| \le 1 \text{ or } |y| \le 1 \text{ or } |z| \le 1\}$
   (i) $I = \{(x, y, z) \mid |x| \le 1 \text{ and } |y| \le 1 \text{ and } |z| \le 1\}$
   (j) $J = \{(x, y, z) \mid |x| \ge 1\}$
   (k) $K = \mathbb{R}^3$ with finitely many points removed.
   (l) $L = \mathbb{R}^3$ with all of the points $\{(a, b, c) \mid a, b, c \in \mathbb{Z}\}$ removed.
   (m) $L = \mathbb{R}^2$ with one single point removed.

The other problems in our "Review Problem Archive" are so similar to problems in your textbook that for brevity's sake I'll list things from the Chapter Review instead of typing them out. The starred problems are more conceptual and could be important on the exam, but don't ignore the more computational problems. You'll have to compute line integrals (of scalar functions and/or vector fields) according to the definition, too.

Chapter 16 Review, pp1136– 1*, 2, 3, 5, 6, 8, 9, 11, 12, 13, 14, 21*, 24*, 37*, 38*.

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