Math 2374

Overview : 2.4, 2.6, Chapter 4, 5, and 7.1-7.2, 8.1

This review sheet is not meant to be your only form of studying. Understanding all the homework problems and lecture material are essential for success in the course. This review sheet only contains the key ideas of these sections.

2.4: Paths and curves.

1. Know that a curve can be parametrized by a function $c(t)$. Also, $c'(t)$ is the velocity of an object at position $c(t)$. Moreover, $c'(t)$ is **tangent vector** to the curve at time t.
2. Be able to compute the tangent line to a curve.

2.6: Gradients and Directional derivatives.

1. The gradient ∇f:
   (a) For scalar-valued function f, the gradient ∇f is like the matrix of partial derivatives Df, except that the gradient is a vector rather than a matrix.
   (b) The gradient is a vector whose magnitude and direction have physical meaning:
   - The gradient points in the direction where f increase most rapidly.
   - the magnitude of the gradient indicates the rate of change in f in that direction.

   (c) For $f : \mathbb{R}^n \to \mathbb{R}$. The gradient ∇f is perpendicular to level sets of f, we can use the gradient to find tangent planes to surfaces.
   For example, when $n = 3$, the tangent plane of the level surface $f(x, y, z) = k$ (k is a constant) at point (x_0, y_0, z_0) is

$$\nabla f(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0.$$

2. The directional derivative:
   (a) The directional derivative $D_v f$ gives the rate of change of f in direction v.
   (b) $D_v f(x) = \nabla f(x) \cdot v$ where v is a **unit vector**.

Chapter 4.

4.1-4.2: Acceleration and Newton's second law and Arc length.

1. Differentiation rules:

$$\frac{d}{dt}[c_1(t) \cdot c_2(t)] = c_1'(t) \cdot c_2(t) + c_1(t) \cdot c_2'(t)$$
$$\frac{d}{dt}[c_1(t) \times c_2(t)] = c_1'(t) \times c_2(t) + c_1(t) \times c_2'(t)$$

2. if $c(t)$ is a vector function such that $\|c(t)\|$ is a constant, then $c'(t)$ is perpendicular to $c(t)$ for all t.
3. (**Newton's second law**) Let $c(t)$ be the distance function of a moving particle of mass m in $\mathbb{R}^3$. Then the velocity $v(t) = c'(t)$ and the acceleration $a(t) = v'(t) = c''(t)$. Newton's second law is

$$(force)\ F = ma.$$

4. (**Arc length**) The (arc) length of the path $c(t) = (x(t), y(t), z(t))$ for $a \le t \le b$ is

$$L = \int_a^b \|c'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt.$$

4.3 : Vector fields.

1. Know how to sketch vector fields.
2. A **flow line** for a vector fields F is a path $c(t)$ such that $c'(t) = F(c(t))$.

4.4: Divergence and curl.

1. Key idea for divergence: measures outflow per unit volume of fluid flow.
2. Key idea for curl: measures rotation of fluid flow.
3. $\text{div}F = \nabla \cdot F$ and $\text{curl}F = \nabla \times F$.
4. (**curl of a gradient**) $\nabla \times (\nabla f) = 0$. See example 11 in page 252.
5. (**divergence of a curl**) $\nabla \cdot (\nabla \times f) = 0$. See example 13 in page 253.

Chapter 5.

5.1-5.5: Double integrals.

1. Key idea: although defined by Riemann sums over rectangles, these integrals can be computed through iterated integrals.
2. Be able to compute bounds for iterated integrals, especially for the different orders of integration.
3. Remember: outer limits must be constant; inner limits can depend only on variables from the outside integral.
4. Finding limits and changing order of integration are easiest if you draw pictures.

Chapter 7.

7.1: the path integral.

1. The path integral of the scalar function f along the path $c(t), a \leq t \leq b$ is defined by

$$\int_c f ds = \int_a^b f(c(t))\|c'(t)\|dt.$$

2. If $f(c)$ is density of wire, then $\int_c f ds$ is mass of wire.
3. If $f = 1$, then $\int_c f ds = \int_c ds = \int_a^b \|c'(t)\|dt$ is length of c.
4. $\int_c f ds$ is independent of parametrization of the curve c.

7.1: Line integrals.

1. The **line integral** of F along the path $c(t), a \leq t \leq b$ is defined by

$$\int_c F \cdot d\mathbf{s} = \int_a^b F(c(t)) \cdot c'(t)dt.$$

2. If F is a force field, then $\int_c F \cdot d\mathbf{s}$ is the work done by the force field on a particle moving along the path c.

3. $\int_c F \cdot d\mathbf{s}$ is independent of parametrization of c, but depends on the direction of c and satisfies

$$\int_{c^-} F \cdot d\mathbf{s} = -\int_c F \cdot d\mathbf{s},$$

where c^- is the opposite path to c.

8.1: Green's Theorem

1. Let $\mathbf{F} = (P, Q)$. Green's theorem is

$$\int_{\partial D} Pdx + Qdy = \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

Here we need a "positively oriented" boundary $C = \partial D$ correctly. The region D must be on your left as you move along C.

2. $\text{Area}(D) = \frac{1}{2}\int_{\partial D} x\,dy - y\,dx.$