Math 1271 - Fall, 2007

FINAL EXAM

T.A.________________________ Signature:________________________

Instructor____________________ Discussion Section _______ I.D. #____

READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 15 pages, including this cover page. Check to see if any are missing. PRINT all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. Books, notes, cells and calculators are NOT PERMISSIBLE. Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 16 machine-graded problems worth 7 points each, for a total of 105 points. There are 8 hand-graded problems, worth a total of 95 points. There is also a question about the textbook at the end of the machine-graded part, that we would like you to answer. The total for the entire exam is 200 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-16):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 17-22):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam. Either the student or the School of Mathematics may for any reason request a regrading of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. THEREFORE, WORK AND ANSWERS MUST BE CLEARLY SHOWN ON THE TEST BOOKLET

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning in your exam.

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1. The equation \( xy^2 + 3xy + x^2 + 5y = 4 \) defines \( y \) implicitly as a function of \( x \). Then \( \frac{dy}{dx} \) is equal to

(A) \( \frac{4 - y^2 - 3y - 2x}{2xy + 3x + 5} \)

(B) \( \frac{y^2 + 3y + 2x}{2xy + 3x + 5} \)

(C) \( -\frac{y^2 + 3y + 2x}{2xy + 3x + 5} \)

(D) \( -\frac{2xy + 3x + 5}{y^2 + 3y + 2x} \)

(E) \( -\frac{2x}{2xy + 3x + 5} \)

2. Find the horizontal and vertical asymptotes for \( f(x) = \frac{(x - 2)^2}{(x + 2)(x - 1)} \)

(A) \( x = -2 \) and \( x = 1 \)

(B) \( x = -2 \) and \( x = 1 \) and \( x = 2 \)

(C) \( x = 2 \) and \( y = 0 \)

(D) \( x = -2 \) and \( x = 1 \) and \( y = 0 \) and \( y = 1 \)

(E) \( x = -2 \) and \( x = 1 \) and \( y = 1 \)
3. Let $f$ be a function whose first derivative is given by $f'(x) = \frac{2x + 3}{(x - 1)^2}$. Then $f$ is decreasing on

(A) $(-\infty, -3/2]$

(B) $(-\infty, -3/2]$ and $[1, \infty)$

(C) $[2, 3]$

(D) $(-\infty, -2/3]$

(E) $(-\infty, -1]$

4. Let $f(x) = x^4 - 6x^2$. Then

(A) $f(x)$ has an absolute minimum at $x = 0$ and no local maxima;

(B) $f(x)$ has absolute minima at $x = \pm \sqrt{3}$ and a local, but not absolute, minimum at $x = 0$;

(C) $f(x)$ has absolute minima at $x = \pm \sqrt{3}$ and a local, but not absolute, maximum at $x = 0$;

(D) $f(x)$ has an absolute minima at $x = \pm \sqrt{3}$ and an absolute maximum at $x = 0$;

(E) $f(x)$ has no local minimum and no local maximum.
5. Let \( f(x) \) be defined by

\[
f(x) = \begin{cases} 
  |x - 2|, & \text{if } x < 3; \\
  (x - 2)^2, & \text{if } 3 \leq x \leq 4; \\
  x - 4, & \text{if } x > 4. 
\end{cases}
\]

Then \( f \) is continuous

(A) except at \( x = 2 \);
(B) except at \( x = 3 \);
(C) except at \( x = 4 \);
(D) except at \( x = 3 \) and \( x = 4 \);
(E) except at \( x = 2, x = 3 \) and \( x = 4 \).

6. The tangent line to the curve \( y = 3x^2 - 3x + 4 \) at the point \((2, 10)\) has equation

(A) \( y - 10 = 13(x - 2) \)
(B) \( y - 10 = 10(x - 2) \)
(C) \( y - 10 = 9(x - 2) \)
(D) \( y - 10 = (6x - 3)(x - 2) \)
(E) \( y - 10 = (3x^2 - 3x + 4)(x - 2) \)
7. The area of a square is increasing at the rate of 6 square inches per second. When a side of the square is 3 inches, the diagonal is increasing at what rate? Express your answer in inches per second.

(A) 1
(B) $2\sqrt{3}$
(C) $3\sqrt{2}$
(D) $\sqrt{2}$
(E) $2\sqrt{2}$

8. \( \frac{d}{dx} \left[ \frac{x^2 - x}{3x + 1} \right] \) is equal to

(A) \( \frac{2x - 1}{3} \)
(B) \( \frac{3x^2 + 2x - 1}{(3x + 1)^2} \)
(C) \( \frac{9x^2 - 4x - 1}{(3x + 1)^2} \)
(D) \( \frac{2x - 1}{(3x + 1)^2} \)
(E) \( \frac{1 - 2x - 3x^2}{(3x + 1)^2} \)
9. If we estimate $\sqrt{8.3}$ by use of linearization or differentials, we get

(A) 2  
(B) 2.3  
(C) 2.25  
(D) 2.025  
(E) 2.027

10. Compute $\lim_{x \to 0} \frac{\sqrt{1 - 8x} - \sqrt{1 - 3x}}{x}$.

(A) 5/2  
(B) -5  
(C) -5/2  
(D) -11  
(E) -11/2
11. Let \( f(x) = 2x^3 - 9x^2 + 12x \). Then

(A) \( f(x) \) has a local maximum at \( x = 1 \), an absolute minimum at \( x = 2 \), and an inflection point at \( x = 3/2 \);

(B) \( f(x) \) has local maxima at \( x = 1 \) and \( x = 3/2 \), and a local minimum at \( x = 2 \);

(C) \( f(x) \) has a local minima at \( x = 1 \) and \( x = 2 \), and an inflection point at \( x = 3/2 \);

(D) \( f(x) \) has inflection points at \( x = 1, x = 2 \) and \( x = 3/2 \);

(E) \( f(x) \) has a local maximum at \( x = 1 \), a local minimum at \( x = 2 \), and an inflection point at \( x = 3/2 \).

12. \( \int_{0}^{\frac{3}{5}} \frac{x}{\sqrt{1 - x^2}} \, dx = \)

(A) \( \sin^{-1}(\frac{4}{5}) = \arcsin(\frac{4}{5}) \)

(B) \( \frac{1}{5} \)

(C) \( \frac{2}{5} \)

(D) \( -\frac{3}{5} \)

(E) \( \frac{4}{5} \)
13. If \( \int_{1}^{\sqrt{3}} \frac{1}{x} \, dx = 3 \), then \( \int_{1}^{a} \frac{1}{x} \, dx = \)

(A) 9  
(B) 6  
(C) 12  
(D) \( \sqrt{3} \)  
(E) cannot be determined from the given information

14. The substitution \( x = u^2 \) turns \( \int_{2}^{3} \tan \sqrt{x} \, dx \) into

(A) \( \int_{\sqrt{2}}^{\sqrt{3}} \tan u \, du \)  
(B) \( \int_{\sqrt{2}}^{\sqrt{3}} 2u \tan u \, du \)  
(C) \( \int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{2} u \tan u \, du \)  
(D) \( \int_{4}^{9} \tan u \, du \)  
(E) \( \int_{4}^{9} 2u \tan u \, du \)
15. Let \( f(x) = \int_{1}^{2x} \sqrt{7t^2 + 8} \, dt \). Then the derivative \( f'(1) = \)

(A) 0
(B) 6
(C) 12
(D) \(\sqrt{15}\)
(E) \(2\sqrt{15}\)
Hand-graded part

16. In the implicit relationship \( x^2 - 9y^2 = 9 \)

A) (20%) Find \( y'' \) as a function of \( x \) and \( y \).

B) (5%) Find \( y'' \) as a function of \( y \) only.
17. The region \( R \) in the \( x - y \) plane bounded by \( y = 0, \ y = e^{-x^2}, \ x = 0, \ x = 5 \) is rotated about the \( y \) axis to give a solid \( S \).

(A) using the method of cylindrical shells, set up an integral (or sum of integrals) to express the volume of \( S \).

(B) using the method of slices (or “disk method”), set up an integral (or sum of integrals) to express the volume of \( S \).

(C) Choose (a) to evaluate the volume of \( S \).
18. Let \( f(x) = x^3 - 3x^2 + 1 \)

On which intervals is \( f(x) \)

a) increasing

b) decreasing

c) concave up

d) concave down

e) sketch the graph of \( f(x) \) in the grid below.
19. Find an approximation to $\sqrt{5}$ using Newton's method. Take $f(x) = x^2 - 5$, and $x_1 = 2$ and find $x_3$. Express the answer as a mixed fraction (like $2 \frac{3}{4}$).
20. Find the area of the rectangle having largest area that can be inscribed in a semicircle of radius $R$. Justify your reasoning.
21. Find the limit, or explain why it does not exist.

a) \( \lim_{x \to 1} \left( \frac{1}{\ln x} - \frac{1}{x - 1} \right) \)

b) \( \lim_{x \to -\infty} \left( \frac{x}{x + 2} \right)^x \)