EX: \[ \lim_{x \to 3^+} \frac{x+e^x}{(3-x)e^x} = \frac{3 + e^3}{e^3} > 0. \]

\[ \lim_{x \to 3^+} \frac{1}{3-x} = -\infty \]

So, \[ \lim_{x \to 3^+} \frac{x+e^x}{(3-x)e^x} = -\infty \]

EX: \[ \lim_{t \to 0} \left( \frac{1}{2t\sqrt{1+2t}} - \frac{1}{2t} \right) \]

\[ = \lim_{t \to 0} \frac{1 - \sqrt{1+2t}}{2t\sqrt{1+2t}} \]

\[ = \lim_{t \to 0} \frac{(1 - \sqrt{1+2t})(1 + \sqrt{1+2t})}{2t\sqrt{1+2t}(1 + \sqrt{1+2t})} \]

\[ = \lim_{t \to 0} \frac{-2t}{2t\sqrt{1+2t}(1 + \sqrt{1+2t})} \]

\[ = \frac{-1}{2} \]
\[ f(x) = \begin{cases} -x^2 + a, & x \leq 2 \\ 6 - x^2, & x > 2 \end{cases} \]

Find \( a \) so that \( f \) is continuous at \( x = 2 \).

\[
\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (-x^2 + a) = -4 + a
\]

\[
\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (6 - x^2) = 2
\]

\[-4 + a = 2.
\]

\[ a = 6. \]
EX: \( g(x) = \frac{x^2 - 2x + 8}{x + 2} \).

(a) Does \( g \) have horizontal asymptotes?

\[
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{x^2 - 2x + 8}{x + 2} = \infty.
\]

\[
\lim_{x \to -\infty} \frac{x^2 - 2x + 8}{x + 2} = \lim_{x \to -\infty} \frac{x - \frac{8}{x}}{1 + \frac{2}{x}} = -\infty.
\]

\( \boxed{\text{No}} \)

(b) Does \( g \) have vertical asymptotes?

Yes. \( x = -2 \).

Check: \( \lim_{x \to -2} \frac{x^2 - 2x + 8}{x + 2} = -\infty \).
§ 3.5 Implicit Differentiation.

The functions we met before can be expressed as \( y = f(x) \), e.g., \( y = \frac{1}{x}, \ y = \sqrt{x^2 + 1} \).

However, some functions are defined implicitly by a relation between \( x \), \( y \) such as

\[
x^2 + y^2 = 25, \quad y = \pm \sqrt{25 - x^2}.
\]

\[
x^3 + y^3 = 6xy.
\]

It is hard to solve an equation for \( y \) in terms of \( x \) in order to find \( \frac{dy}{dx} \). So we need a new method called implicit differentiation to find \( \frac{dy}{dx} \).

**EX:** \( x^2 + y^2 = 25 \). Find \( \frac{dy}{dx} \).

**Ans:**

\[
\frac{d}{dx} (x^2 + y^2) = \frac{d}{dx} (25)
\]

\[
x^2 + \frac{d}{dx} (y^2) = 0
\]

\[
2x + \frac{d}{dx} (y^2) = 0
\]

\[
\frac{d}{dx} (y^2) = \frac{d}{dy} (y^2) \frac{dy}{dx} \quad \text{(by chain rule)}
\]

\[
= 2y \frac{dy}{dx}
\]

So,

\[
2x + 2y \frac{dy}{dx} = 0
\]

\[
\Rightarrow \frac{dy}{dx} = -\frac{x}{y}.
\]
EX. \( x^3 + y^3 = 6xy \). Find \( \frac{dy}{dx} \).

**Ans:** \[
\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(6xy)
\]

\[
3x^2 + 3y^2 \cdot y' = 6y + 6x \cdot y'
\]

\[
(3y^2 - 6x) \cdot y' = 6y - 3x^2
\]

\[
y' = \frac{6y - 3x^2}{3y^2 - 6x}
\]

\[
y' = \frac{2y - x^2}{y^2 - 2x}
\]

(b) Find the tangent line to this curve at (3,3).

**Ans:** Tangent line equation:

\[
y - 3 = y'(3) \cdot (x - 3)
\]

\[
y'(3) = \left. \frac{2y - x^2}{y^2 - 2x} \right|_{(3,3)} = \frac{6 - 9}{9 - 6} = -1
\]

\[
y - 3 = -1(x - 3)
\]