1. Which, if any, of the following statements are true? (Choose all)

(a) If $f'$ is continuous on $[a, b]$, then $\int_a^b f'(x) \, dx = f(a) - f(b)$. **Need to flip those arrows to be correct.**

(b) If $A(x) = \int_a^x f(t) \, dt$ and $f(t) = 2t - 3$, then $A$ is a quadratic function.

(c) If $f$ is continuous on $[0, 1]$ and $\int_0^1 f(x) \, dx = 0$, then $f(x) = 0$ for $0 \leq x \leq 1$.

(d) If $f$ is an odd function and $\int_0^4 f(x) \, dx = 10$, then $\int_{-4}^4 f(x) \, dx = 20$.

(e) $\int_{-1}^1 (x^2 + \cos x) \, dx = 2 \int_0^1 (x^2 + \cos x) \, dx$.

(f) $\int_0^a f(x) \, dx = f(-x)$

(g) $\int_0^a f(x) \, dx = 0$.

(h) $\int_{-1}^1 \sin x \, dx = 2 \int_0^1 \sin x \, dx$.

(i) $(\sin x)^2 + \cos x$ is even function.

(j) $(\sin x)^2 + \cos x$ is even function.

#1 ANSWER: b, e.

(e) is incorrect the way it is; the correct formulation is $\int_a^b f'(x) \, dx = f(b) - f(a)$ which is true by Fund. Thm. of Calculus (since $f$ is an antiderivative of $f'$ and the additional assumption of $f'$ being cont. is stated).
2. Using the Midpoint Rule with \( n = 4 \), approximate \( \int_{-4}^{4} (2x + x^3) \, dx \).

\[
\Delta x = \frac{4 - (-4)}{4} = 2
\]

\[
\begin{array}{cccc}
-4 & -2 & 0 & 2 & 4 \\
\hline
\end{array}
\]

\( f(x) = 2x + x^3 \).

\[
M_4 = f(-3) \cdot 2 + f(-1) \cdot 2 + f(1) \cdot 2 + f(3) \cdot 2
\]

\[
= (-6 + 9) \cdot 2 + (-2 + 1) \cdot 2 + (2 + 1) \cdot 2 + (6 + 9) \cdot 2
\]

\[
= 2 \left( 3 - 1 + 3 + 15 \right)
\]

\[
= 2 \cdot 20
\]

\[
= 40
\]
3. Suppose that $f(x)$ and $g(x)$ are integrable and that
\[
\int_{-2}^{2} f(x) \, dx = 4, \quad \int_{2}^{5} f(x) \, dx = 3, \quad \int_{-2}^{5} g(x) \, dx = 2, \quad \text{and} \quad \int_{2}^{5} g(x) \, dx = -1.
\]

Find the following integrals.

(a) \[\int_{5}^{2} f(x) \, dx = -\int_{2}^{5} f(x) \, dx = -3 \quad \#\]

(b) \[
\int_{-2}^{5} (f(x) + 2g(x)) \, dx = \int_{-2}^{2} f(x) \, dx + 2 \int_{2}^{5} g(x) \, dx
\]
\[= (\int_{-2}^{2} f(x) \, dx + \int_{2}^{5} f(x) \, dx) + 2 \quad (2)\]
\[= 4 + 3 + 4 \quad \#\]
\[= 11 \quad \#\]

(c) \[
\int_{-2}^{2} -3g(x) \, dx.
\]
\[\int_{-2}^{5} g(x) \, dx = \int_{-2}^{2} g(x) \, dx + \int_{2}^{5} g(x) \, dx
\]
\[= \int_{-2}^{2} g(x) \, dx + (-1) \quad (2)\]
\[\text{Hence,} \quad \int_{2}^{5} g(x) \, dx = 3\]
\[\int_{-2}^{2} -3g(x) \, dx = -9 \quad \#\]
4. Evaluate the integral. Both (a), (b) only if your midterm
(covers sect. 5, 5

(a) \[ \int \cos(x) e^{\sin x} \, dx. \]

Let \( u = \sin x \)
\( du = \cos x \, dx \)

\[ \int e^u \, du \]
\[ = e^u + C \]
\[ = e^{\sin x} + C \]

(b) \[ \int_0^2 \frac{4x}{(x^2 + 1)^2} \, dx \]

Let \( u = x^2 + 1 \)
\( du = 2x \, dx \)

\[ \int_0^2 \frac{2}{(x^2 + 1)^2} \, dx \]
\[ = \int_1^5 \frac{2}{u^2} \, du \]
\[ = (-2u^{-1}) \bigg|_1^5 \]
\[ = -2 \left( \frac{1}{5} - 1 \right) = \frac{8}{5} \]
5. For a particle moving along a line, the acceleration function (in m/s²) and the initial velocity are given by \( a(t) = 6t \), \( v(0) = -12 \).

(a) Find the displacement during the time interval \( 0 \leq t \leq 3 \).

\[
V = a \Rightarrow V(t) = 3t^2 + (-12) = 3t^2 - 12
\]

Since \( v(0) = -12 \),

\[
\text{displacement} = \int_0^3 V(t) \, dt = \int_0^3 (3t^2 - 12) \, dt
\]

\[
= \left[ t^3 - 12t \right]_0^3
\]

\[
= 27 - 36
\]

\[
= -9 \text{ (m)}
\]

(b) Find the total distance traveled during the time interval \( 0 \leq t \leq 3 \).

\[
\int_0^3 |V(t)| \, dt
\]

\[
= \int_0^3 |3t^2 - 12| \, dt
\]

\[
3t^2 - 12 = 0 \Rightarrow t = \pm 2
\]

\[
\int_0^3 |3t^2 - 12| \, dt = \int_0^2 (3t^2 - 12) \, dt + \int_2^3 (3t^2 - 12) \, dt
\]

\[
= \left[ t^3 - 12t \right]_0^2 + \left[ t^3 - 12t \right]_2^3
\]

\[
= (-8 + 24) + (-8 + 24) - (8 - 24)
\]

\[
= 16 + 16
\]

\[
= 32 \text{ (m)}
\]
6. Sketch the region enclosed by the curves \( y = x \), \( y = 2\sqrt{x} \) and find its area.

Find intersection points of \( y = x \) and \( y = 2\sqrt{x} \). Then

\[
x = 2\sqrt{x}
\]

\[
\Rightarrow x^2 = 4x
\]

\[
\Rightarrow x(x - 4) = 0
\]

\[
\Rightarrow x = 0, 4
\]

\[
\text{Area} = \int_{0}^{4} \left( 2\sqrt{x} - x \right) \, dx
\]

\[
= \left[ \left( \frac{4}{3} x^{\frac{3}{2}} - \frac{1}{2} x^2 \right) \right]_0^4
\]

\[
= \frac{4}{3} \left( 4 \right)^{\frac{3}{2}} - \frac{1}{2} (16) - 0
\]

\[
= \frac{32}{3} - 8
\]

\[
= \frac{8}{3}
\]
7. A rancher wants to construct a rectangular corral. He also wants to divide the corral by a fence parallel to one of the sides. He has 540 feet of fence. What are the dimensions of the corral of largest area he can enclose?

\[ 3x + 2y = 540 \implies y = \frac{540 - 3x}{2} \]

Area = \( A = xy \)

\[ A(x) = x \left( 270 - \frac{3}{2}x \right) \]

\[ A'(x) = 270 - \frac{3}{2}x - \frac{3}{2}x = 0 \]
\[ x = 90 \quad , \quad y = 270 - \frac{3}{2}(90) = 135 \]

Since \( A'(x) < 0 \) if \( x > 90 \), by 1st derivative test, \( x = 90 \) is a maximum.

28. (10 pts) A storage room with a square base is to hold 3,000 cubic feet. Material for the floor and top of the room costs $3 per square foot; material for the four sides costs $8 per square foot. Find the dimensions of the room so as to minimize the cost of the material.

\[ V = x^2 y = 3000 \implies y = \frac{3000}{x^2} \]

\[ A = 3 \left( 2x^2 \right) + 8 \left( 4xy \right) \]

\[ A(x) = 6x^2 + 32x \left( \frac{3000}{x^2} \right) \]

\[ A'(x) = 12x - \frac{32 \cdot 3000}{x^2} = 0 \]
\[ 12x^3 - 32 \cdot 3000 = 0 \]
\[ x^3 = 8000 \]
\[ x = 20 \quad \text{feet} \quad , \quad y = \frac{3000}{400} = \frac{15}{2} \quad \text{feet} \]

Since \( A'(x) > 0 \) if \( x > 20 \), by 1st derivative test, \( x = 20 \) is a minimum. The room should be 20 feet long and 15 feet tall to minimize the cost.
More Practice Problems Solutions

Math 1271

Compute \( f'(x) \) if

(a) (5 points)

\[
f(x) = \int_{x}^{16} \sin(t^2) \, dt
\]

\[
f'(x) = -\sin(\tan(x^2))
\]

(b) (5 points)

\[
f(x) = \int_{0}^{2\pi} t^2 e^{t^2} \, dt
\]

\[
f'(x) = 2 \left(2\pi\right)^{\frac{3}{2}} e^{(2\pi)^2}
\]

\[= 8\pi^2 e^{4\pi^2}
\]

(c) (10 points)

\[
f(x) = \int_{x}^{2} e^{t^2} \, dt
\]

\[
f'(x) = 2x e^{x^2} - e^{x^2}
\]
Find indefinite integrals in the general form.

(a) \( \int \frac{\arctan x}{1+x^2} \, dx \)

\[
\begin{align*}
U &= \tan^{-1} x \\
\, du &= \frac{1}{1+x^2} \, dx \\
\int \, du &= \frac{1}{2} u^2 + C \\
&= \frac{1}{2} (\tan^{-1} x)^2 + C
\end{align*}
\]

(b) \( \int (x+1)\sqrt{x^2+2x} \, dx \)

\[
\begin{align*}
U &= x^2 + 2x \\
\, du &= (2x + 2) \, dx \\
&= 2(x+1) \, dx \\
\int \sqrt{u} \, \frac{du}{2} &= \frac{1}{3} U^{\frac{3}{2}} + C \\
&= \frac{1}{3} (x^2 + 2x)^{\frac{3}{2}} + C
\end{align*}
\]
\[ \int_{2}^{5} \frac{dx}{\sqrt{x-1}} = \text{(Only if your midterm goes through section 5.5)} \]

(A) 2 

(B) 1 

(C) 3 

(D) -1 

(E) -2 

\[ u = x - 1 \]
\[ du = dx \]

\[ \int_{2}^{5} \frac{dx}{\sqrt{x-1}} = \int_{1}^{4} \frac{1}{\sqrt{u}} \, du \]

\[ = \int_{1}^{4} u^{-\frac{1}{2}} \, du \]

\[ = 2 u^{\frac{1}{2}} \bigg|_{1}^{4} \]

\[ = 2 (2 - 1) \]

\[ = 2 \]
The point on the graph of \( y = x^2 \) which is closest to the point \((16, \frac{1}{2})\) has coordinates

A. \((1, 1)\)

B. \((2, 4)\)

C. \((3, 9)\)

D. \((\frac{3}{2}, \frac{9}{4})\)

E. \((\frac{9}{4}, \frac{81}{16})\)

To minimize the distance between

\((x, x^2)\) and \((16, \frac{1}{2})\)

\[
A(x) = \sqrt{(x-16)^2 + (x^2 - \frac{1}{2})^2}
\]

Let \( f(x) = A^2(x) = (x-16)^2 + (x^2 - \frac{1}{2})^2 \)

\[
f'(x) = 2(x-16) + 2(x^2 - \frac{1}{2}) 2x
\]

\[
= 2x - 32 + 4x^3 - 2x
\]

\[
= 4x^3 - 32 = 0
\]

\[
x = 2
\]

\[
y = 4
\]
\( f(x) \) is a function such that \( f''(x) = -\frac{1}{x^2} + 4 \sin x \). In addition it is known that \( f(1) = 0 \) and \( f'(1) = 1 \).

a. Find \( f'(x) \).

(a) \( f'(x) = x^{-1} - 4 \cos x + C \)

\[ f'(1) = 1 - 4 \cos(1) + C = 1 \]

\[ C = 4 \cos(1) \]

\( f(x) = x^{-1} - 4 \cos x + 4 \cos(1) \)

(b) \( f(x) = \ln |x| - 4 \sin x + 4 \cos(1) x + D \)

\[ f(1) = 0 - 4 \sin(1) + 4 \cos(1) + D = 0 \]

\[ D = 4 \sin(1) - 4 \cos(1) \]

\( f(x) = \ln |x| - 4 \sin x + 4 \cos(1) x + (4 \sin(1) - 4 \cos(1)) \)

b. Find \( f(x) \).
At midnight, ship A is 20 miles North of the origin, and sailing South along the y-axis at 15 mph. Ship B is 22 miles West of the origin, and is sailing East along the x-axis at 25 mph.

a) Express the distance between the two ships as a function of time. (Take \( t = 0 \) at midnight, and measure \( t \) in hours)

\[
A^2 = (-10 - 15t)^2 + (22 - 25t)^2
\]

b) What is the closest distance of approach of the two ships, and when does it occur?

\[
A(t) = (20-15t)^3 + (22-25t)^2
\]

\[
\frac{d}{dt} A(t) = 2(20 - 15t)(-15) + 2(22 - 25t)(-25)
\]

\[
= -600 + 450t + 1100 + 1250t = 0
\]

\[
t = \frac{1700}{1750} = 1 \text{ (hr)}
\]

By the derivative test, when \( t = 1 \), it is the closest distance.

\( f(x) \) is a function such that \( f''(x) = x + \cos x \). In addition it is known that

\[
f(0) = 0, \quad f'(0) = 1 \text{ and } f''(0) = -1;
\]

a) Find \( f''(x) \)

\[
f''(x) = \frac{1}{2} x^2 + \sin x + C.
\]

\[
f''(0) = 0 + C = -1
\]

\[
f''(x) = \frac{1}{2} x^2 + \sin x - 1
\]

b) Find \( f'(x) \).

\[
f'(x) = \frac{1}{6} x^3 - \cos x - x + D
\]

\[
f'(0) = 0 - 1 - 0 + D = 1 \Rightarrow D = 2
\]

\[
f'(x) = \frac{1}{6} x^3 - \cos x - x + 2
\]

c) Find \( f(x) \).

\[
f(x) = \frac{1}{24} x^4 - \sin x - \frac{1}{2} x^2 + 2x + E
\]

\[
f(0) = E = 0
\]

\[
f(x) = \frac{1}{24} x^4 - \sin x - \frac{1}{2} x^2 + 2x
\]
A rectangular garden of 200 square meters in area is to be fenced off against rabbits. Find the dimensions that will require the least amount of fencing if one side of the garden is protected by a barn (and hence does not need a fence).

\[ xy = 200 \text{ (m}^2) \]

\[ A = 2x + y \]

\[ A(x) = 2x + \frac{200}{x} \]

\[ A'(x) = 2 - \frac{200}{x^2} = 0 \]

\[ 2x^2 - 200 = 0 \]

\[ x = 10 \text{ (m)} \]

\[ y = \frac{200}{10} = 20 \text{ (m)} \]

Since \[ A'(x) > 0 \text{ if } x > 10 \]

\[ A'(x) < 0 \text{ if } x < 10 \]

by 1st derivative test,

\[ x = 10 \text{ (m)} \], \[ y = 20 \text{ (m)} \]

that requires the least amount of fencing.
No vertical asymptotes since \( f(x) \) is defined and continuous for all \( x \).

(Only if your midterm includes sect. 4.5)

Consider the function \( f(x) = 3x^{2/3} - x \).

(a) Find (if any): the \( x \) and \( y \)-intercepts, all horizontal/vertical asymptotes and all critical numbers.

\[ y \text{-intercept: set } x = 0: \quad f(0) = 3(0) - 0 = 0, \]  
so the graph passes through the origin \((0,0)\).

\[ x \text{-intercepts: set } y = f(x) \text{ equal } 0: \]
\[ 3x^{2/3} - x = 0 \Rightarrow x^{2/3}(3 - x^{1/3}) = 0 \]
\[ x^{2/3} = 0, \text{ i.e., } x = 0, \]
or
\[ 3 - x^{1/3} = 0, \text{ i.e., } \]
\[ x^{1/3} = 3, \]
\[ x = 27. \]

(b) Determine where the function is increasing and decreasing.

\[ f'(x) = 3 \cdot \frac{2}{3}x^{-1/3} - 1 = \frac{2}{\sqrt[3]{x}} - 1 \]

\[ f'(x) = 0: \quad \frac{2}{\sqrt[3]{x}} = 1 \Rightarrow \frac{3}{\sqrt[3]{x}} = 2 \Rightarrow x = 8. \]

\[ f'(x) \text{ does not exist when } x = 0 \]

So critical numbers are \( x = 0, x = 8 \).

\[ f''(x) = \frac{2}{x^{4/3}} - \frac{1}{x^{2/3}} \]

\[ f''(27) = \frac{2}{3} - 1 < 0 \]

\[ f''(-1) = \frac{2}{-1} - 1 < 0 \]

\[ f'(1) = 2 - 1 > 0 \]

\( f'(x) > 0 \)
Continued

No inflection points, since an inflection point is one where it turns from being concave downward to concave upward or vice versa.

(a) Find the inflection points.

\[ f''(x) = \left(2x^{-\frac{2}{3}} - 1\right)' = 2\left(-\frac{1}{3}\right)x^{-\frac{4}{3}} = -\frac{2}{3} \cdot \frac{1}{x^{\frac{4}{3}}} \]

\[ x^{\frac{4}{3}} = \sqrt[3]{x^4} > 0 \text{ for all } x \neq 0, \]

hence \( \frac{1}{x^{\frac{4}{3}}} > 0 \text{ for all } x \neq 0. \)

Hence \( f''(x) = -\frac{2}{3} \cdot \frac{1}{x^{\frac{4}{3}}} \) for all \( x \neq 0. \)

L undefined when \( x = 0. \)

L does not exist.

Hence \( f(x) \) is concave downward on \((-\infty, 0)\) and on \((0, \infty)\).

(d) Determine where the function is concave upwards and concave downwards.
Use the definition of the definite integral to evaluate \( \int_0^3 (x^2 - 2) \, dx \).

Do not use FTC2 or any other method. These summation formulae may be helpful:

\[
\sum_{i=1}^n 1 = n, \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \left( \frac{n(n+1)}{2} \right)^2.
\]

\[
\int_0^3 (x^2 - 2) \, dx = \lim_{n \to \infty} R_n \rightarrow f(x) = x^2 - 2.
\]

\[
= \lim_{n \to \infty} \sum_{i=1}^n f(x_i) \Delta x, \text{ where } x_i \text{ are the right end points.}
\]

\[
a = 0, \quad b = 3, \quad \Delta x = \frac{b-a}{n} = \frac{3}{n},
\]

\[
x_i = a + i \Delta x = 0 + i \cdot \frac{3}{n} = \frac{3i}{n}
\]

\[
\lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^n \left( \frac{9i^2}{n^2} - 2 \right) = \lim_{n \to \infty} \frac{3}{n} \left( \frac{\sum_{i=1}^n \frac{9i^2}{n^2} - \sum_{i=1}^n 2}{n} \right)
\]

\[
= \lim_{n \to \infty} \frac{3}{n} \left( \frac{\frac{9}{n^2} \sum_{i=1}^n i^2 - (2+2+\ldots+2)}{n} \right)
\]

\[
= \lim_{n \to \infty} \frac{3}{n} \left( \frac{\frac{9}{n^2} \sum_{i=1}^n i^2 - 2n}{n} \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{\frac{27}{n^3} \sum_{i=1}^n i^2 - 6}{6} \right) = \lim_{n \to \infty} \left( \frac{\frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - 6}{6} \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{\frac{9}{2} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n} - 6}{6} \right) = \lim_{n \to \infty} \left( \frac{\frac{9}{2} \cdot \left(1 + \frac{1}{n}\right)^3 - 6}{6} \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{9}{2} \cdot \left(1 \cdot \frac{1}{n}\right)^3 - 6 \right)
\]

\[
= 9 - 6 = 3
\]
8) If Newton's method is used to approximate the unique solution of $4 \ln x = x$ in the interval $1 < x < 2$, then the initial approximation $x_0 = 1$ would lead to a first approximation $x_1 =$

(A) 1
(B) $\frac{5}{4}$
(C) $\frac{3}{2}$
(D) $\frac{4}{3}$
(E) 2

Let $f(x) = 4 \ln x - x$, $f'(x) = \frac{4}{x}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{4 \ln x_n - x_n}{\frac{4}{x_n}}$$

$$= x_n - \frac{4 \ln x_n - x_n}{4} - 1$$

So

$$x_1 = x_0 - \frac{4 \ln x_0 - x_0}{\frac{4}{x_0}} - 1$$

$$= 1 - \frac{4 \ln 1 - 1}{\frac{4}{1}} - 1$$

$$= 1 - \frac{-1}{3} = 1 + \frac{1}{3} = \frac{4}{3}$$
A billboard containing 32 square feet of area is to be designed. The advertiser wants margins of 2 feet on the left and right side of the advertisement and margins of one foot on the top and bottom. What dimensions should the billboard have in order to maximize the space for advertisement?

(A) 16 \times 2 \text{ feet}
(B) 4 \times 8 \text{ feet}
(C) 32 \times 1 \text{ feet}
(D) 4\sqrt{2} \times 4\sqrt{2} \text{ feet}
(E) none of the above

So space for advertisement is

\[ f(x) = \left( \frac{32}{x} - 2 \right) (x - 4) \]

\[ f'(x) = -\frac{32}{x^2} (x - 4) + \left( \frac{32}{x} - 2 \right) \]

\[ = -\frac{32}{x^2} + \frac{128}{x^2} + \frac{32}{x} - 2 = 0 \]

\[ \frac{128}{x^2} = 2, \quad x^2 = 64 \]

\[ x = 8 \]

So \( f'(x) > 0 \) when \( 0 < x < 8 \),

\( f'(x) < 0 \) when \( x > 8 \),

So by first derivative test, maximum area for advertisement is when \( x = 8 \).

Height = \( \frac{32}{x} = 4 \)
Compute the following integrals. Some are definite.

(a) \[
\int_{0}^{16} (6\sqrt{t} + 15\frac{4}{\sqrt{t}}) \, dt
\]
\[
= 6 \left[ \frac{t^{3/2}}{3/2} \right]_0^{16} + 15 \left[ \frac{t^{5/4}}{5/4} \right]_0^{16} + C
\]
\[
= 6 \left( \frac{16^{3/2}}{3/2} \right) + 15 \left( \frac{16^{5/4}}{5/4} \right) + C
\]
\[
= (4 \cdot 32) + (12 \cdot 16) = 128 + 192 = 320
\]

(b) \[
\int_{1}^{2} \frac{(x-3)(x+3)}{x^2} \, dx
\]
\[
= \int_{1}^{2} \frac{x^2-9}{x^2} \, dx = \int_{1}^{2} \left( 1 - \frac{9}{x^2} \right) \, dx
\]
\[
= \left[ x + \frac{9}{x} \right]_{1}^{2} = \left( \frac{16}{2} + \frac{9}{2} \right) - \left( 1 + \frac{9}{1} \right)
\]
\[
= \frac{7}{2}
\]

\[\boxed{20}\]
Let
\[ x^2 \cos(\pi x) = \int_{-1}^{x} f(t) \, dt \]
where \( f \) is a continuous function. Find \( f \left( \frac{1}{2} \right) \).

\[
\frac{d}{dx} \left( x^2 \cos(\pi x) \right) = \frac{d}{dx} \int_{-1}^{x} f(t) \, dt = f(x)
\]

\[
f(x) = \frac{d}{dx} \left( x^2 \cos(\pi x) \right) = 2x \cos(\pi x) - x^2 \left( \sin(\pi x) \right) \pi
\]

\[
f \left( \frac{1}{2} \right) = 2 \cdot \frac{1}{2} \cos \left( \frac{\pi}{2} \right) - \frac{1}{4} \left( \sin \left( \frac{\pi}{2} \right) \right) \pi
\]

\[
= -\frac{\pi}{4}
\]

Find the derivative:

\[
\frac{d}{dx} \left( \tan^2 x \right) \frac{1}{\sqrt{x}} = \left[ \sin \left( (\tan^2 x)^2 \right) \right] (\tan^2 x) \,'

\[
- (\sin(\sqrt{x})^2) (\sqrt{x}) '
\]

\[
= \left[ \sin \left( \tan^4 x \right) \right] (2 \tan x) \sec^2 x - (\sin x) \cdot \frac{1}{2\sqrt{x}}
\]

\[
= 2 \tan x \sec^2 x \sin(\tan^2 x) - \frac{1}{2\sqrt{x}} \sin x
\]
Multiple choices.

(1). Let \( f(x) = \int_{2}^{\sqrt{2x} \cdot \sqrt{3t^2 + 6}dt} \). Then \( f'(2) = \)

(A) \( \sqrt{2} \)

(B) 6

(C) \( \sqrt{30} \)

(D) 0

(E) \( 2\sqrt{15} \)

\[ f'(x) = \sqrt{3 \left( \sqrt{2x} \right)^2 + 6} \cdot \sqrt{2} \]

\[ f'(2) = \sqrt{6 \cdot (2^2) + 6} \cdot \sqrt{2} \]

\[ = \sqrt{30} \cdot \sqrt{2} = \sqrt{2(15)} \cdot \sqrt{2} \]

\[ = \sqrt{2} \sqrt{15} \cdot \sqrt{2} = 2\sqrt{15} \]

Which of the following limits represent the definite integral \( \int_{0}^{2} x^2 \sec(x)dx \)?

(A) \( \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{i}{n} \right)^2 \sec \left( \frac{i}{n} \right) \)

(B) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n} \left( \frac{i}{n} \right)^2 \sec \left( \frac{i}{n} \right) \)

(C) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n} \left( \frac{2i}{n} \right)^2 \sec \left( \frac{2i}{n} \right) \)

(D) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n} \left( \frac{i}{n} \right)^2 \sec \left( \frac{i}{n} \right) \)

(E) None of the above.

\[ \Delta x = \frac{b-a}{n} = \frac{2}{n} \]

\[ R_n = \sum_{i=1}^{n} f(x_i) \cdot \Delta x \]

\[ x_i = a + i \cdot \Delta x \]

\[ x_i = \frac{2i}{n} \]

\[ \int_{0}^{2} x^2 \sec(x)dx = \lim_{n \to \infty} R_n \]

\( \boxed{22} \)
Using Newton’s method to approximate a solution to the equation \( f(x) = 0 \), where \( f \) is a differentiable function with \( f'(x) \neq 0 \): if \( x_n \) has been determined, then \( x_{n+1} \) is given by the following formula:

(A) \( x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \)
(B) \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \)
(C) \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \)
(D) \( x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \)
(E) \( x_{n+1} = x_n + f(x_n) \cdot f'(x_n) \)

\( C \) denotes an arbitrary real number. The general antiderivative of the function

\[
f(x) = \cos x + e^{2x}
\]

is which of the following?

(A) \( F(x) = \sin x + 2e^{2x} + C \)
(B) \( F(x) = -\sin x + 2e^{2x} + C \)
(C) \( F(x) = -\sin x + \frac{1}{2}e^{2x} + C \)
(D) \( F(x) = \sin x + \frac{1}{2}e^{2x} + C \)
(E) None of the above

\( (A) \quad F'(x) = \cos x + 4e^{2x} \quad \text{No!} \)
\( (B) \quad F'(x) = -\cos x + 4e^{2x} \quad \text{No!} \)
\( (C) \quad F'(x) = -\cos x + e^{2x} \quad \text{No!} \)
\( (D) \quad F'(x) = \cos x + e^{2x} \quad \text{Yes!} \)
1. Multiple choices.

(1) Let \( f(x) = \int_2^x \sqrt{7t^2 + 8} \, dt \), what is \( f'(2) \)?

(A) 0  \[ f'(x) = \sqrt{7x^2 + 8} \]

(B) 2  \[ f'(2) = \sqrt{7(4)+8} = \sqrt{36} = 6 \]

(C) 4  \[ \]

(D) 6  \[ \]

(E) 6

2. The substitution \( x = u^2 \) turns \( \int_2^3 \cot \sqrt{x} \, dx \) into

(A) \( \int_2^3 \cot u \, du \)

(B) \( \int_{\sqrt{2}}^{\sqrt{3}} 2u \cot u \, du \)

(C) \( \int_2^4 2u \cot u \, du \)

(D) \( \int_2^3 u \cot u \, du \)

(E) \( \int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{2}u \tan u \, du \)

3. If \( \int_1^e \frac{1}{x} \, dx = 6 \) then \( \int_1^e \frac{1}{x} \, dx = \)

(A) 6  \( \ln|\alpha| \int_{\sqrt{a}}^{\sqrt{a}} = \ln|\sqrt{a}| - \ln1 = \ln\sqrt{a} = 6 \)

(B) 9  \( \)

(C) 12

(D) \( \sqrt{3} \)

(E) Cannot be determined from the given information
2. All four parts of this problem refer to the function 

\[ f(x) = 2 + 3x^2 - x^3. \]

(a) Find the critical number(s) of \( f \).

\[ f'(x) = 6x - 3x^2 = 0 \]

\( f \) is polynomial;

\( x(6 - 3x) = 0 \)

So \( f'(x) \) exists for all \( x \)

\( x = 0, x = 2 \)

\[ x = 0, 2 \],

(b) Find the interval(s) of increase of \( f \), and find the interval(s) of decrease of \( f \).

\( f' < 0 \quad f' > 0 \quad f' < 0 \)

\[-1 \quad \text{test} \quad 0 \quad \text{test} \quad 2 \quad \text{test} \quad 3 \]

\( (-\infty, 0) \text{ dec.}, \quad (0, 2) \text{ inc.}, \quad (2, \infty) \text{ dec.} \)

(c) For each critical number, determine whether it is the location of a local maximum, local minimum, or neither. Be sure to state the name of a “test” in your reasoning.

At \( 0 \) is loc. min. by first deriv. test since

\( f'(x) < 0 \quad \text{for} \quad x < 0, x \text{ near } 0 \),

\( f'(x) > 0 \quad \text{for} \quad x > 0, x \text{ near } 0 \).

At \( 2 \) is loc. max by first deriv. test,

since \( f'(x) > 0 \quad \text{for} \quad x < 2, x \text{ near } 2 \),

\( f'(x) < 0 \quad \text{for} \quad x > 2, x \text{ near } 2 \).

(d) Find the inflection point(s) of \( f \).

\[ f''(x) = 6 - 6x \quad \Rightarrow \quad x = 1 \]

\[ f'' > 0 \quad f'' < 0 \]

concave upward on \( (-\infty, 1) \) \( x = 1 \) inflection pt.

concave down on \( (1, \infty) \) turns from conc. up to conc. down.
Evaluate the following integrals.

(a) \[ \int \frac{(x+2)(x-2)}{\sqrt{x}} \, dx \quad \text{and} \quad \int \frac{(x+2)(x-2)}{\sqrt{x}} \, dx \]

\[
= \int \frac{x^{5/2} - 4}{\sqrt{x}} \, dx = \int (x^{3/2} - 4x^{-1/2}) \, dx \\
= \frac{x^{5/2}}{\frac{5}{2}} - 4 \cdot \frac{x^{1/2}}{\frac{1}{2}} + C = \frac{2}{5}x^{5/2} - 8x^{1/2} + C
\]

(b) \[ \int \frac{\sin x}{1 + \cos^2 x} \, dx \quad \text{(only if your midterm goes through sect. 5.5)} \]

\[ \cos x = u, \quad \Rightarrow \quad du = (-\sin x) \, dx \Rightarrow \quad -du = \sin x \, dx \]

\[ \int \frac{1}{1 + \cos^2 x} \, (\sin x \, dx) = \int \frac{1}{1 + u^2} \, (-du) = -\int \frac{1}{1 + u^2} \, du \]

\[\downarrow \quad \arctan(u) + C \quad \Rightarrow \quad -\arctan(u) + C \]

\[= -\arctan(\cos x) + C \]

Evaluate the integral by using Properties of the Integral (big box on p. 379 in the book) and interpreting the integrals in terms of area:

(c) \[ \int_{-5}^{5} (20 + \sin(x^3) + \sqrt{25 - x^2}) \, dx \]

\[= \int_{-5}^{5} 20 \, dx + \int_{-5}^{5} \sin(x^3) \, dx + \int_{-5}^{5} \sqrt{25 - x^2} \, dx \]

\[= 20 \int_{-5}^{5} \, dx + \int_{-5}^{5} \sin(x^3) \, dx \]

\[\text{Rectangle, height}=20, \quad \text{width}=10 \]

\[\sqrt{20(10)} = 200 \]

\[\text{\sin(x^3) is an odd function} \]

\[\therefore \quad \int_{-5}^{5} \sin(x^3) \, dx = 0 \]

\[\int_{-5}^{5} \sqrt{25 - x^2} \, dx \]

\[= \frac{1}{2} \pi (5^2) = 25 \pi \]
The expression
\[
\left( \frac{1}{1+\frac{1}{10}} + \frac{1}{1+\frac{2}{10}} + \frac{1}{1+\frac{3}{10}} + \ldots + \frac{1}{1+\frac{10}{10}} \right) \cdot \frac{1}{10}
\]
is a Riemann sum approximation of
(a) \( \int_0^1 \frac{1}{x} \, dx \)
(b) \( \int_0^{\frac{1}{1+x}} \frac{1}{x} \, dx \) but not of \( \int_0^{\frac{2}{x}} \frac{1}{x} \, dx \)
(c) \( \int_1^{\frac{1}{1+x}} \frac{1}{x} \, dx \) but not of \( \int_1^{\frac{1}{1+x}} \frac{1}{x} \, dx \)
(d) both of \( \int_0^{\frac{1}{1+x}} \frac{1}{x} \, dx \) and \( \int_1^{\frac{1}{2}} \frac{1}{x} \, dx \)
(e) \( \int_1^{\frac{2}{10x}} \frac{1}{x} \, dx \)

Let \( f(x) = \frac{1}{x} \), \( 1 \leq x \leq 2 \).

Then work out \( R_{10} : \Delta x = \frac{2-1}{10} = \frac{1}{10} \),
\[
R_{10} = \sum_{i=1}^{10} f(x_i) \cdot \frac{1}{10} = \sum_{i=1}^{10} \frac{1}{x_i} \cdot \frac{1}{10} = \frac{10}{\sum_{i=1}^{10} x_i + \frac{1}{10}}
\]
which equals

Let \( f(x) = \frac{1}{1+x} \), \( 0 \leq x \leq 1 = b \), \( \Rightarrow \Delta x = \frac{1}{10} \),
\[
R_{10} = \sum_{i=1}^{10} f(x_i) \cdot \frac{1}{10} = \sum_{i=1}^{10} \frac{i}{10} \cdot \frac{1}{10} = \frac{10}{\sum_{i=1}^{10} i + \frac{1}{10}} \]
So both \( f(x) = \frac{1}{x} \) for \( 1 \leq x \leq 2 \)
\[
f(x) = \frac{1}{1+x} \text{ are possible.}
\]
Find the point \( A \), in the first quadrant, and on the parabola \( y = 4 - x^2 \), such that the rectangle with vertices at \( A \) and at the origin, and sides on the axes, and parallel to the axes, has largest possible area.

The vertices \((0,0)\) (origin) and \( A \) are to be diagonally opposite.

So the shaded area is to be largest possible.

The area of the shaded rectangle is

\[
s(x) = x(4-x^2), \quad \text{so} \quad s'(x) = 4 - x^2 + x(-2x)
\]

\[
= 4 - x^2 - 2x^2 = 4 - 3x^2
\]

\[4 - 3x^2 = 0\]

\[3x^2 = 4 \Rightarrow x = \pm \frac{2}{\sqrt{3}}, \quad \frac{2}{\sqrt{3}} < 2, \quad \frac{2}{\sqrt{3}} > 1\]

We need \( x > 0 \), so

\[x = \frac{2}{\sqrt{3}}\]

\[y = 4 - x = 4 - \left(\frac{2}{\sqrt{3}}\right)^2 = 4 - \frac{4}{3} = \frac{8}{3}\]

So the desired point \( A = \left(\frac{2}{\sqrt{3}}, \frac{8}{3}\right)\).

We use the first derivative test to show that \( A \) yields the maximum area:

\[s'(x) = 4 - 3x^2\]

\[s'(x) = 0 \quad \text{at} \quad x = \frac{2}{\sqrt{3}}\]

\[s'(x) < 0 \quad \text{for} \quad x > \frac{2}{\sqrt{3}}\]

\[28\]