Math 4428. MATHEMATICAL MODELING. Spring 2017

Homework #3 (due on Wednesday, April 5, at the beginning of class). 50 points are divided between problems 1–4. In your solutions, whether or not you are using a computer software, you need to justify each step.

#1. (10 points.) Solve Problem 4.4.8 (a) on p.135 of the textbook.

#2. (12 points.) Consider the matrix
\[ A = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -4 & -1 \\ 0 & 5 & 0 \end{pmatrix}. \]

(a) Find eigenvalues and independent eigenvectors of \( A \).
(b) Find the real general solution of the system
\[ \frac{dx}{dt} = \begin{pmatrix} x' \\ x' \\ x' \end{pmatrix} = A \cdot \begin{pmatrix} x \\ x \\ x \end{pmatrix}. \]
(c) Discuss stability of the equilibrium point \((0,0,0)^T\).

#3. (12 points.) Solve Problem 5.4.10(b,c) on p.167 of the textbook.

#4. (16 points.) For the system
\[ \begin{align*}
    x'_1 &= -2x_1 + x_2 + x_1^2 + x_1x_2, \\
    x'_2 &= x_1 - 2x_2 + x_2^2 + x_1x_2,
\end{align*} \]

(a) discuss the type and stability of the equilibrium point \((0,0)\);
(b) find all other equilibrium points and discuss their type and stability;
(c) by introducing the function \( u(t) = x_1(t) + x_2(t) \), show that if \( x_1(0) + x_2(0) > 1 \), then the solution \((x_1(t), x_2(t))\) becomes unbounded as \( t \to t_0 \) for some \( t_0 > 0 \) (depending on \( x_1(0) + x_2(0) \)).