Homework #1 (due on Wednesday, February 19).

50 points are divided between 5 problems. In your solutions, whether or not you are using a computer software (e.g. Mathematica), you need to justify each step.

#1. (12 points.) Reformulate the pig market problem (Example 1.1 on p.4 in the textbook), assuming that a pig gains $\alpha$ pounds per day, where $0 \leq \alpha \leq 10$.

(a). Find the best time $x$ to sell the pig and the maximum profit $P$, as functions of $\alpha$.
(b). Compute the sensitivities $S(x, \alpha)$ and $S(P, \alpha)$ at the point $\alpha = 5$.
(c). Now suppose that the cost to keep is proportional to the weight $w$ of the pig, with the initial cost of $0.45$ a day, i.e. it is $0.45w/200$ dollars a day. Under this assumption, find the best time $x$ to sell the pig and the maximum profit $P$ for $\alpha = 5$.

#2. (8 points.) Solve Problem 1.4.9 (c) on p.18 in the textbook for general $n$, and apply the results to parts (a) and (b) in the case $n = 5,000$.

#3. (10 points.) In Example 2.1 on p.21 in the textbook, consider an imaginary situation with 0.3 and 0.4 cents being replaced by 3 and 4 cents correspondingly. This means that instead of (2.2)–(2.3) you now have to maximize

$$f(x_1, x_2) = (339 - 0.01x_1 - 0.03x_2)x_1 + (399 - 0.04x_1 - 0.01x_2)x_2 - (400,000 + 195x_1 + 225x_2)$$

over the set $S = \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0\}$.

#4. (10 points.) (a). Find the maximal possible volume $V$ of a cylinder

$$C = \{(x_1, x_2, x_3) : x_1^2 + x_2^2 \leq r^2, 0 \leq x_3 \leq h\}$$

with the given total surface area $S$.

(b). Same question for a parallelepiped

$$P = \{(x_1, x_2, x_3) : 0 \leq x_1 \leq a, \leq x_2 \leq b, 0 \leq x_3 \leq c\}$$

instead of a cylinder $C$.

#5. (10 points.) Solve Problem 2.4.6 (a) and (b) on p.52 in the textbook.