Problem 1. Solve the initial value problem
\[ x \cdot \frac{dy}{dx} + \left(1 + \frac{x \cdot \cos x}{\sin x}\right)y = 0, \quad y \left(\frac{\pi}{2}\right) = 2. \]

Solution. This is a separable equation, which can be rewritten as follows:
\[ \frac{dy}{y} + \left(\frac{1}{x} + \frac{\cos x}{\sin x}\right)dx = 0. \]

By integrating this expression, we get
\[ \ln |y| + \ln |x| + \ln |\sin x| = \text{const}, \quad \text{and} \quad y \cdot x \cdot \sin x = c = \text{const}. \]

The constant \( c = \pi \) is derived from the initial condition. The answer is
\[ y = \frac{\pi}{x \cdot \sin x}, \quad 0 < x < \pi. \]

Problem 2. Find the general solution of the equation
\[ \frac{dy}{dx} + \frac{\cos x \cdot y}{\sin x} = \frac{x}{\sin x}. \]

Solution. The corresponding linear equation
\[ \frac{du}{dx} + \frac{\cos x \cdot u}{\sin x} = 0 \Rightarrow \frac{du}{u} + \frac{\cos x \cdot u}{\sin x} = 0 \]
\[ \Rightarrow \ln |u| + \ln |\sin x| = \text{const} \Rightarrow u \cdot \sin x = \text{const}. \]

One can take \( u(x) = 1/\sin x \) and substitute \( y = uv \) into the original equation. We get
\[ \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx} + \frac{\cos x \cdot uv}{\sin x} = \frac{x}{\sin x} \Rightarrow \frac{1}{\sin x} \cdot \frac{dv}{dx} = \frac{x}{\sin x} \Rightarrow v = \frac{x^2}{2} + \text{const}. \]

The answer is
\[ y = uv = \frac{x^2 + c}{2 \sin x} \text{ in any interval } (n\pi, (n+1)\pi), \text{ where } n \text{ is an integer}. \]

Problem 3. Find the general solution of the equation
\[ \frac{dy}{dx} = \frac{x^2 - 3y^2}{2xy}. \]

Solution. This is a homogeneous equation:
\[ \frac{dy}{dx} = \frac{1}{2} \cdot \left(\frac{y}{x}\right)^{-1} - \frac{3}{2} \cdot \frac{y}{x}. \]
By substitution $v = y/x$, $y = xv$, we obtain

$$
v + x \cdot \frac{dv}{dx} = \frac{1}{2v} - \frac{3v}{2} \implies \frac{x dv}{dx} = \frac{1-5v^2}{2v} \implies \frac{2v}{5v^2-1} = \frac{dx}{x} \implies \frac{1}{5} \cdot \ln |5v^2-1| = - \ln |x| + \text{const} \implies 5v^2-1 = \frac{c}{x^5}.
$$

The answer is $(5y^2 - x^2)x^3 = c \ (x \neq 0, \ y \neq 0)$.

**Problem 4.** Solve the second order equation with given initial conditions:

$$2y'' - 7y' + 3y = 0; \quad y(0) = 0, \quad 2y''(0) - y'(0) = 15.$$

**Solution.** The characteristic equation

$$2r^2 - 7r + 3 = (2r-1)(r-3) = 0 \quad \text{has roots} \quad r_1 = 3, \ r_2 = 1/2.$$

The general solution is $y = c_1 e^{3x} + c_2 e^{x/2}$. From $y(0) = 0$ it follows $c_1 = -c_2 = c$. Now

$$y = c \left( e^{3x} - e^{x/2} \right), \quad y' = c \left( 3e^{3x} - \frac{1}{2} e^{x/2} \right), \quad y'' = c \left( 9e^{3x} - \frac{1}{4} e^{x/2} \right),$$

and

$$(2y'' - y')(x) = 15c \cdot e^{3x} = 15c = 15 \quad \text{at the point} \quad x = 0.$$

Hence $c = 1$, and the answer is $y = e^{3x} - e^{x/2}$.