Fall 2016. Math 5615H.
Homework 3 (due on Wednesday, October 5).

1. (A variant of Exercise 19 in Chapter 1). Let $a = (a_1, \ldots, a_k)$ be a point in $\mathbb{R}^k$ with coordinates $a_1 = 8, a_2 = a_3 = \cdots = a_k = 0$. Find $c \in \mathbb{R}^k$ and $r > 0$ such that $|x - a| = 3|x|$ for $x \in \mathbb{R}^k$ if and only if $|x - c| = r$.

2. Let $x$ any $y$ be vectors in $\mathbb{R}^k$. Show that $y$ is uniquely represented in the form $y = a + b,$ where $a, b \in \mathbb{R}^k$ satisfy $a = \alpha x$ for some real $\alpha$, and $b \cdot x = 0$. Verify this fact for $x = (1, 1, 1)$ and $y = (1, 2, 3)$ in $\mathbb{R}^3$.

3. Let $A$ be a nonempty set in $\mathbb{R}^k$. Define $d(x) := \inf \{|x - a| : a \in A\}$. Show that $|d(x) - d(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}^k$.

4. Show that $(1 + h)^n \geq 1 + nh$ for all natural $n$ and real $h$ such that $|h| \leq 1$. (This is Problem 7(a) on p. 22.) Using this result, show that $a_1 \leq a_2 \leq a_3 \leq \cdots$, where $a_n = \left(1 + \frac{1}{n}\right)^n$.

*Hint.* Consider $a_n/a_{n-1}$.

5. Evaluate $\cos\frac{\pi}{15} \cdot \cos\frac{2\pi}{15} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15}$.

*Warning.* The answer is a rational number!